

Rent-Seeking and Capital Accumulation

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Abstract

We develop a simple way of incorporating rent-seeking activities into the standard neoclassical model of capital accumulation. We use the model to explain stylized facts in development in the presence of unproductive activities. We also use the model to explain a number of relevant issues in development economics.

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1 Introduction

It is well understood that institutional, political and other such factors ought to be considered in economic growth and development. In particular the fact that some activities might not generate socially valued output is bound to have an impact on the long run performance of economies. The accepted models of economic growth only deal with productive activities, though. This paper presents a framework that addresses this issue. We develop a simple way of incorporating unproductive activities in the standard neoclassical model of capital accumulation. We show that the model is consistent with what can be called the stylized facts regarding rent-seeking and growth, and that it can be used to analyze a number of important issues in development economics.

The focus of the paper is on the model itself. Below we will discuss the modelling procedure in some detail. Before we do that, we would like to point out some of the results delivered by the model. First of all, welfare in the economy is inversely related to the technology of unproductive

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activities. As unproductive activities become relatively more efficient, productive factors move towards those activities and away from productive activities, and the final result is a reduction in the welfare of the representative agent.¹ Since the technology of unproductive activities is largely determined by the efficiency of the institutional set in preventing such activities, we have that a deterioration of the institutional set reduces welfare (and this is the first stylized fact that the model delivers).

The other two stylized facts are the desirability of market power in rent-seeking activities (see Murphy, Shleifer and Vishny (1993)) and the fact that unproductive activities help the predicting power of the neoclassical model (see Parente, Rogerson and Wright (2000)). In fact, in our model a monopolist rent-seeker will do less harm to the economy than a competitive rent-seeking sector, and we present a calibration exercise showing that the model fits the data quite well.

On top of those three stylized facts the model delivers some additional results. There is an “endogenous TFP” perspective built in the model: as productive factors move between sectors in the economy the measured TFP changes. That is, since TFP is measured as a residual after considering that all productive factors are allocated in productive activities, the fact that some factors are allocated in unproductive activities shows up in TFP estimates. During transitional dynamics, the reallocations of productive factors generate a transition path for TFP as well. Also, the model provides a rationale to the poor performance of sub-Saharan countries: one can have an economy whose capital stock increases and per capita GDP decreases, which is what has been taking place in some African countries. And finally we can use the model to explain the failure in foreign aid in helping the receiving country (basically because the aid goes to rent-seekers and does not add to the productive capacity of the economy).

The model is simply the neoclassical growth model with the addition of a second sector, the unproductive sector. The contribution lies on how we introduce such sector. We do so by using an ‘aggregate rent-seeking technology’, that translates the unproductive sector’s output into a number between 0 and 1, representing the fraction of the productive sector’s output that is captured by the unproductive sector. We present a set of properties that such a function ought to satisfy and we show that these properties are sufficient to close the model. In particular, no functional form is needed to solve the model. As it is the case with production functions, functional forms are only necessary for some applications (and our function does have a ‘Cobb-Douglas-like’ counterpart).

There are two economic decisions: static and dynamic. The static is the factor allocation problem (for a given level of productive factors), and the dynamic is the consumption-investment

¹Although this is an expected result, it is not necessarily true that long run per capita GDP decreases as productive factors move into unproductive activities. What happens is that during the transition consumption decreases so much that it overcomes the (possible) long run increase in income.

allocation problem. In both cases an equilibrium is defined and shown to exist and be unique. The effect of the institutional efficiency can also be disentangled into static and dynamic parts. For a given level of productive factors the institutional efficiency determines the amount of resources employed in the rent-seeking sector. This is similar to Gordon Tullock’s idea and we call it Tullock effect. Moreover, institutional efficiency also generates a dynamic effect, that of a distortion in capital accumulation. This is the usual effect of a distortion and we call it Harberger effect.² These two effects summarize the workings of the model. The above mentioned effect of institutional efficiency on welfare can be divided into two effects, that correspond to Tullock and Harberger effects. The unambiguous result means that the Tullock effect dominates the Harberger effect when the latter happens to be of opposite sign (the Tullock effect is always of the same sign: the worse the institution set is, the more is captured by the rent-seeking sector. The Harberger effect can be of the opposite sign when rent-seeking is capital intensive). It follows that the fact that productive resources are employed in unproductive activities is the main cause of welfare being reduced by inefficient institutions.³

For the calibration exercise we use data on per capita income and a measure of institutional efficiency both from Hall and Jones (1999) to calibrate two unobservable parameters: one is the Jacobian of the transformation of Hall and Jones’s estimate of institutional efficiency into our parameter θ ; the other is the curvature of our ‘aggregate rent-seeking technology’ function. Under the assumptions that Hall and Jones’s relation between output and institutional efficiency is a good one, and also that rent-seeking accounts for 10% of US’s GDP,⁴ we are able to calibrate those unobservable parameters. The model fits the data quite well. The calibration exercise illustrates that a monopolist rent-seeker is significantly better than a competitive rent-seeking industry to the economy.

The paper is organized as follows. Section 2 relates our work with the previous literature. The model is presented in Section 3. In addition to presenting the assumptions behind the aggregate rent-seeking technology, we define the static and dynamic equilibria and also show their existence and uniqueness. Section 4 presents the comparative statics results, and also the properties of the

²We named the two effects Tullock and Harberger because they resemble the Tullock/Harberger debate of the social costs of monopoly. Harberger (1954) pointed out that the cost of the monopoly is the deadweight loss it generates, and found out that this loss is small. Tullock (1967) replied saying that the monopolist captures part of consumers’ surplus, and hence that real resources would be employed to capture these economic rents, so the cost of a monopoly is much larger than the deadweight loss it generates. Hence, our Tullock effect measures the resources used to capture rents, and our Harberger effect measures the usual cost of inefficient institutions (the dynamic distortion). Posner (1975) evaluated empirically Harberger and Tullock effects for a monopoly in a partial equilibrium framework.

³When both sectors operate under the same technology, the Harberger effect is zero; given that it is always dominated by the Tullock effect and it is only non zero when technologies differ, we may say that it is of second order of importance in welfare terms. Hence the driving force is the distortion in factor allocation (Tullock effect).

⁴See Anderson (1999).

transition dynamics. The stylized facts are presented in Sections 5, 6 and 7. Section 8 presents the additional results mentioned above.

2 Relation to Other Works

Although there has been some interest in the literature on rent-seeking and unproductive activities, there does not exist a systematic literature on the subject. There are contributions on this issue coming from macroeconomics, political economy, international trade, common pool problems, and economic history. In this short section, without been comprehensive, we survey these contributions and show how our work relates to them.

Murphy, Shleifer and Vishny (1993) and Acemoglu (1995) build simple static models of factor allocation in presence of competitive rent-seeking. Their models produce multiple equilibria which should be contrasted with our uniqueness result.⁵ Our assumptions on the aggregate rent-seeking technology guarantee that an inflow of productive factors into the rent-seeking activity reduces this activity's profitability more than the reduction on the profitability of the productive activity. As we argued above, this choice reflects our belief on the relative importance of fundamental (including the institutional set) vis-a-vis initial conditions/coordination failures in understanding issues of economic development.

Tullock (1980), Skaperdas (1992), Hirshleifer (1995), and Grossman and Kim (1995) present models in which a fixed number of individual or bands or groups (usually 2) fight for a slice of a pie, which in some cases is endogenously determined by the production decision of the contenders.⁶ The lack of free entry in the rent-seeking activity means that in equilibrium rents are not fully dissipated. As it was the case with Murphy, Shleifer and Vishny (1993) and Acemoglu (1995), these contributions assume one productive factor, usually a linear production function, and do not consider capital accumulation.

Krueger (1974) and Bhagwati and Srinivasan (1980) present models of rent-seeking in a international trade economic environment (some form of the H.O.V. model). In this literature many of the results rest on the specific interaction among the unproductive activity and other distortions related to international trade. In particular, the unproductive activity might be welfare improving, which is not the case in our setup.

Tornell and Velasco (1992), Benhabib and Rustichini (1996), and Grossman and Kim (1996) present models in which capital accumulation takes place in a dynamic game framework. A given

⁵See footnote ??.

⁶Grossman and Kim (1995) considered that in addition to the two usual activities in this literature - production and predation - there is another activity, protection.

number of agents (two or more) face the strategic choice of how much production to appropriate, which might generate less incentives for production and capital accumulation. The explicit game-theoretic formulation used in those papers is to be contrasted with our assumption of perfect competition, where a large number of rent-seeking firms compete for the appropriation of sector 1's output. Strategic considerations are summarized by the aggregate rent-seeking technology, the Contest Success Function, and by the free entry condition (see section 2). As a result, our formulation is simpler, and, since those models use a single factor technology with constant returns to scale (basically a variant of the *AK* model), it seems to us that our model is the first attempt to incorporate rent-seeking in the neoclassical model of capital accumulation. Moreover, the first two models do not take into consideration the resources employed in the appropriation of sector 1's output (the Tullock effect), which turns out to be the most important effect of the model (see section 5).

The original contribution of Tullock (1967) is taken as the background of our formalization. Eric Jones (1988) provided a very illuminating account of the world economic history in terms of a struggle between rent-seeking and productive activities.⁷ North (1990, 1994), Baumol (1990), Olson (1992), Murphy, Shleifer and Vishny (1993) among others were also instrumental in shaping the hypothesis that institutions form the fundamental structure of incentives that eventually drives all results in a market economy. One of the purposes of setting up the present model was to provide a formalization of these ideas under a general and standard macroeconomic framework.

3 The Model

The model presented in this paper can be viewed as a simple extension of the neoclassical model of capital accumulation. In that model there is just one good produced by a constant returns to scale technology that employs capital and labor, whose services are rented by a representative consumer to the firms. The representative consumer makes her intertemporal decision optimally taking into account the income stream she will receive from her renting of those services. Institutions are usually introduced, in a macroeconomic setup, as a wedge between what firms produce and the income they earn. That is, output of the firms is summarized by an aggregate production function,

⁷The following passage nicely summarizes his argument:

“Economic history may be thought of as a struggle between a propensity for growth and one for rent-seeking, that is, for someone improving his or her position, or a group bettering its position, at the expense of the general welfare. (...) Whenever conditions permitted, that is, when rent-seeking was somehow curbed, growth manifested itself.” (Jones, 1988)

$F(K, L)$, and firms' income is given by a fraction of that output, $(1 - \tau)F(K, L)$. The 'tax rate' τ represents any sort of distortion that might characterize the economy, which could be a tax itself. In general, it can be identified with the efficiency of the institutional background of the economy. The simple extension considered here is to give a specific formulation for the 'tax rate' τ .

In particular, we assume that there exists another sector in the economy, called the unproductive sector (also the rent-seeking sector, or sector 2). Like the productive sector (sector 1), it combines capital and labor to produce an output, but this output is not another good. It is a service, a transfer service. That is, an effort to confiscate goods produced in sector 1. The more service is produced, the larger the amount of goods that gets transferred toward sector 2. Calling Y_1 and Y_2 the output levels of sectors 1 and 2 respectively, the idea above can be stated as follows: sector 1 keeps $(1 - \tau(Y_2))Y_1$ and sector 2 is able to confiscate $\tau(Y_2)Y_1$ goods from sector 1, where τ is an increasing function of the transfer services, Y_2 . Formally, the burden imposed by the rent-seeking sector on the productive sector is a negative externality, which would not emerge if property rights were fully enforced.

The function τ will be fully derived and characterized below (it will be denoted by g to reserve the symbol τ for a bona fide tax rate). This function g is the main analytical contribution of the model.

3.1 Aggregate Rent-Seeking

From the technological point of view, the major distinction between the productive activity and the unproductive one is that in order to 'produce' unproductive output inputs of capital and labor services, *and output*, are required. The productive activity, on the other hand, requires only capital and labor services. Let G be the total amount of output which is extracted from the productive sector by the unproductive sector. We assume that $G = G(\theta Y_2, Y_1)$, where the function G is homogeneous of the first degree, Y_2 is the total output of transfer services, and θ describes the quality of the institutional set. A high (low) θ represents a bad (good) institutional background. We view θ as a measure of 'total factor productivity' (TFP) of sector 2, and hence θ enters as an argument of G multiplying Y_2 . From the homogeneity of G we write

$$G = g\left(\theta \frac{Y_2}{Y_1}\right) Y_1 = g(\theta y^R) Y_1, \quad (1)$$

where $y_2 \equiv \frac{Y_2}{L}$, $y^R \equiv \frac{y_2}{y_1}$, and $g\left(\theta \frac{Y_2}{Y_1}\right) \equiv G\left(\frac{\theta Y_2}{Y_1}, 1\right)$. The function g is the share of the output of the productive sector that is extracted by the unproductive sector.⁸ As anticipated above the share g is precisely the tax rate τ that firms in sector 1 take as given (see below):

$$g(\theta y^R) = \tau.$$

Our formulation states that the aggregate rent-seeking technology, g , must be a function of the relative output $\frac{y_2}{y_1}$ multiplied by an institutional variable θ . We assume that $g(0) = 0$ and $g'(x) > 0$, for any $x \geq 0$, and that $\lim_{x \rightarrow \infty} g(x) = 1$. These are natural assumptions. We also assume that g satisfies the following four assumptions. Let $\bar{\alpha}_g \in (0, 1)$ be given, and define $\alpha_g(x) \equiv x \frac{g'(x)}{g(x)} \frac{1}{1-g(x)}$. Let also α_{1L} be sector 1's labor share on income.

Assumption 1 $\lim_{x \rightarrow 0} g'(x) = \infty$ and $g''(x) < 0$.

Assumption 2 $0 < \alpha_g(x) \leq \bar{\alpha}_g$.

Assumption 3 $\bar{\alpha}_g < \alpha_{1L}$.

Assumption 4 $g(x) = \frac{m(x)}{1+m(x)}$ for some m s.t. $m'(x) > 0$, $m''(x) < 0$, $m(0) = 0$, $\lim_{x \rightarrow 0} m'(x) = \infty$.

Assumption 1 is the standard Inada condition plus strict concavity, and Assumption 2 provides an upper bound for $\alpha_g(x)$. These two assumptions ensure uniqueness of equilibrium, which reflects our view that development issues are to be explained by differences on the fundamentals of economies, and not by coordination failures.⁹ Observe that if α_g happens to be constant then integrating $\alpha_g(x)$ yields

$$g(x) = \frac{x^{\alpha_g}}{1 + x^{\alpha_g}}, \tag{2}$$

which is one candidate for a functional form¹⁰ for g . Assumption 3 is needed for long-run stability and is only used in that part of the model. It ensures saddle-path stability of the dynamic system.

⁸The function G plays, in the context of rent seeking, the role of the matching function in the equilibrium unemployment literature. (See Mortensen and Pissarides, 1994.) There g is the rate that seekers of job position meet vacancies; here g is the rate that the seekers of rents exploit the productive sector. Although one activity, job search, is productive and the other, rent-seeking, is not, the formal properties of the function g are the same.

⁹Of course, multiplicity of equilibria can be introduced by relaxing the strict concavity assumption. This would lead to the issue of indeterminacy of equilibrium and of coordination failures. Such phenomena belong, in our view, to short to medium run macroeconomic theories. In the very long run, what matters is the more fundamental properties of an economy. (Evidently, coordination failures, history dependence, or political economy issues can help understanding why fundamental properties of two economies differ. See for instance Engerman and Sokoloff (1997).)

¹⁰Observe the analogy with the Cobb-Douglas functional form.

Finally, Assumption 4 ensures uniqueness of equilibrium in the monopoly formulation (Section 6), and it is needed only for that section. The main model is solved for a generic function g satisfying Assumptions 1,2, and 3.

3.2 Firms

Sector $j = 1, 2$ consists of N_j identical firms operating under the same technology. Sector 1's output is the homogeneous good in the economy, and sector 2's output is a transfer service (an effort to capture sector 1's output). Firm i in sector j combines capital, K_{ji} , and labor, L_{ji} , according to a constant returns to scale technology F_j to produce output Y_{ji} . That is, $Y_{ji} \equiv F_j(K_{ji}, L_{ji})$.

Part of what a firm in sector 1 produces is captured by the firms operating in sector 2. That is, the unproductive activity acts like a tax rate τ on the output of each firm in sector 1: firm i in that sector keeps only $(1 - \tau)Y_{1i}$ of its output. Under perfect competition, firm i 's program is to

$$\max_{K_{1i}, L_{1i}} (1 - \tau)Y_{1i} - r_1K_{1i} - w_1L_{1i}, \quad (3)$$

where r_1 and w_1 are the rental and wage rates prevailing in sector 1.

For firms in sector 2, instead of a tax on their output we have a variable q that determines the amount of output that each unit of transfer effort is able to capture. Clearly q is endogenously determined in equilibrium. But each firm in sector 2 takes q as given. It follows that each firm in sector 2 solves

$$\max_{K_{2i}, L_{2i}} qY_{2i} - r_2K_{2i} - w_2L_{2i}, \quad (4)$$

where r_2 and w_2 are the rental and wage rates prevailing in sector 2.

The first order conditions are given by

$$r_1 = (1 - \tau)f_1'(k_1) \quad (5)$$

$$w_1 = (1 - \tau) [f_1(k_1) - k_1f_1'(k_1)] \quad (6)$$

$$r_2 = qf_2'(k_2) \quad (7)$$

$$w_2 = q [f_2(k_2) - k_2f_2'(k_2)], \quad (8)$$

where $f_j \equiv F_j(k_j, 1)$ and $k_j \equiv \frac{K_{ji}}{L_{ji}}$, $j = 1, 2$.

Sector j 's total output is given by $Y_j = \sum_{i \in N_j} Y_{ji}$. In per capita terms, $y_j \equiv \frac{Y_j}{L} = l_j f_j(k_j)$, where L is the population and $l_j \equiv \frac{1}{L} \sum_{i \in N_j} L_{ji}$ is sector j 's labor share.

3.3 Static Equilibrium

The static equilibrium is an equilibrium in the allocation of productive factors between the two sectors, for given levels of productive factors and institutional efficiency (k and θ). We use the underlying two-sector structure of the model to define such equilibrium. The idea is that each combination of output levels of both sectors determines a marginal rate of transformation and is in turn determined by the latter. The equilibrium is a fixed point of this mutual determination. In what follows we first define such an equilibrium and show that it exists and is unique.

3.3.1 Definition and Existence

The static equilibrium is a consequence of factor mobility. With mobility and an interior solution, it must be the case that $r_1 = r_2$ and $w_1 = w_2$, otherwise all productive factors would be allocated in just one of the industries. From the two-sector general equilibrium model, we know that

$$r_j = p_j f'_j(k_j) \quad (9)$$

$$w_j = p_j [f_j(k_j) - k_j f'_j(k_j)], \quad (10)$$

where p_j is the price of sector j 's output, $j = 1, 2$. Comparing (5), (6), (7), and (8) to (9) and (10), we get that the situation where $p_1 = 1 - \tau$ and $p_2 = q$ is an equilibrium. This is what we call static equilibrium. That is, using $p = \frac{p_2}{p_1}$ to denote the Marginal Rate of Transformation (MRT), the underlying two-sector structure allows us to write $y_i(p, k)$ as sector i 's static supply function.¹¹

The equilibrium is given by $\frac{q}{1-\tau} = p$. In other words, as factors are reallocated between industries p shows how y^R changes and $\frac{q}{1-\tau}$ shows how the relative output in terms of goods changes. In equilibrium those changes must be equal.

Symmetry among firms implies that the fraction captured by sector 2 should be equal to $\frac{qY_2}{Y_1}$, which means that $\tau = qy^R$. Using $g = \tau$ we get

$$p = \frac{q}{1-\tau} = \frac{g(\theta y^R(p, k))}{y^R(p, k)} \frac{1}{1-g(\theta y^R(p, k))} \equiv H(p, k, \theta), \quad (11)$$

so that the static equilibrium is defined as self-determining MRT given by this fixed-point.

If $p > H(p, k, \theta)$ ($p < H(p, k, \theta)$) then factors will move towards sector 2 (sector 1), reducing (increasing) p and increasing (reducing) H because sector 2 (sector 1) pays relatively more. In equilibrium, factor prices are equalized and there is no further factor reallocation.

¹¹Appendices A.1 and A.2 provide a short review of the static two-sector general equilibrium model. See chapter 1 of Kemp (1969) for a more thoroughly presentation.

Let \underline{p} and \bar{p} be the prices under which the economy is specialized in sector 1 and 2 respectively.¹²

Proposition 1 *The short-run equilibrium exists and is unique.*

Proof. Assumption 2 ensures that $H : [\underline{p}, \bar{p}] \rightarrow R_+$ is monotone in p (hence we get uniqueness), and the Inada conditions of Assumption 1 ensure existence. See Appendix B.1 for the details. ■

3.4 Alternative Derivation of the Static Problem - Some Microstructure

Above we presented an aggregate approach to the short-run supply side of the economy. Since the idea of the paper is to introduce unproductive activities in a macro framework, we view such approach as appropriate. Nevertheless, below we present one way of providing some microstructure to the rent-seeking sector. The idea is to show that the static equilibrium defined above is compatible with a rich microstructure. We are confident that one could provide a much richer microstructure (incorporating some political economy ideas) that would generate the same static equilibrium.

3.4.1 Unproductive Firm

Assume that the quantity of goods that a firm in sector 2 expropriates from sector 1 is a share of the total booty G in (1). In particular, we assume that this share is of the additive Contest Success Function (CSF) form,¹³ so that it can be written as $\frac{h(\theta Y_{2i})}{\sum_{j \in N_2} h(\theta Y_{2j})}$. That is, firm i will fight for a share of G and the success of such a fight will be determined by the CSF. We assume that $h(0) = 0$ and $h'(x) > 0$, for any $x \geq 0$. We will make one further assumption.

Assumption 5 There exists a unique \bar{x} such that $\arg \max_x \frac{h(x)}{x} = \bar{x}$.

In other words, there exists one, and just one, optimal scale for each firm in sector 2. Observe that Assumption 5 does not require uniqueness of a point where marginal returns equal average returns, it only posits the existence of just one point that maximizes average returns.

Now assume that the analysis is made on the limit in which there are many firms in each sector so that the Dixit-Stiglitz (1977) assumption of discharging terms that depend on $\frac{1}{N_1}$ or $\frac{1}{N_2}$ from the first order conditions can be made.^{14,15} Consequently, firm i 's program becomes

¹²For a given level of factor endowment k , $p \geq \bar{p}(k)$ ($p \leq \underline{p}(k)$) means that the economy is specialized in the production of rent-seeking services (sector 1's good). Note that $\bar{p}'(k) \geq 0$ and $\underline{p}'(k) \leq 0$ as $k_1 \geq k_2$.

¹³Tullock (1980) introduced the CSF in the theory of rent-seeking. Hirshleifer (1989) named it and established its main properties and Skaperdas (1996) axiomatized the additive CSF.

¹⁴The attentive reader will have noticed that we have already make this assumption in deriving (5) and (6): since $\tau = g\left(\theta \frac{Y_2}{Y_1}\right)$, to take τ as given amounts to assume away the effect of a particular firm in sector 1 on Y_1 .

¹⁵Consequently, we are assuming that the optimum size of a firm in sector 2, \bar{x} , is small enough such that in equilibrium N_2 is large.

$$\max_{K_{2i}, L_{2i}} \frac{h(\theta Y_{2i})}{\sum_{j \in N_2} h(\theta Y_{2j})} g \left(\theta \frac{\sum_{j \in N_2} Y_{2j}}{Y_1} \right) Y_1 - r_2 K_{2i} - w_2 L_{2i}, \quad (12)$$

and the first order conditions are

$$r_2 = \frac{\theta h'(\theta Y_{2i})}{\sum_{j \in N_2} h(\theta Y_{2j})} g(\theta y^R) f_2'(k_2) Y_1, \quad (13)$$

$$w_2 = \frac{\theta h'(\theta Y_{2i})}{\sum_{j \in N_2} h(\theta Y_{2j})} g(\theta y^R) [f_2(k_2) - k_2 f_2'(k_2)] Y_1. \quad (14)$$

We can think of firm i 's program as a two-stage program: the aggregate proceeds of the unproductive industry are determined in the first stage, and then distributed among the firms in that industry in the second stage. Such a perspective reveals the lack of property rights on the proceeds of the rent-seeking industry: any given firm has the potential to access the whole aggregate booty.¹⁶

3.4.2 Free Entry

In keeping with the competitive paradigm, equilibrium within each sector is achieved when each firm makes zero profit. It follows from (5) and (6) that $\pi_{1i} = 0$ for any $i \in N_1$ (hence N_1 is indeterminate). For sector 2, substituting (13) and (14) into (12) yields

$$\pi_{2i} = \frac{h(\theta Y_{2i})}{\sum_{j \in N_2} h(\theta Y_{2j})} g(\theta y^R) Y_1 \left(1 - \theta Y_{2i} \frac{h'(\theta Y_{2i})}{h(\theta Y_{2i})} \right), \quad (15)$$

which is not necessarily zero. Here is where Assumption 5 plays a role. Setting $Y_{2i} = \frac{\bar{x}}{\theta}$ in (15) yields $\pi_{2i} = 0$ (because $h'(\bar{x}) = \frac{h(\bar{x})}{\bar{x}}$), so this level of Y_{2i} for every firm in sector 2 is an equilibrium with free entry. It is unique by hypothesis.¹⁷

Substituting the free entry condition $\pi_{2i} = 0$ into (13) and (14), it follows that a symmetric

¹⁶In the contest of rent-seeking this formalization is equivalent to the idea of a competitive labor market: any firm and any worker have potentially access to the whole market. Such idea is quite useful, since one does not have to deal with the complicated dynamics of search and matching that is likely to describe real world employer-employee relationships. We use the same principle here to describe the unproductive industry.

¹⁷It is also a Nash equilibrium for the game played by the firms in sector 2. That is, assume each firm plays \bar{x} and consider firm i contemplating playing $x \neq \bar{x}$ instead. Straightforward computations yield $\pi_{2i} = \frac{h(x)}{(N_2-1)h(\bar{x})+h(x)} g \left(\theta \frac{y_2}{y_1} \right) Y_1 - r_2 K_{2i} - w_2 L_{2i}$. Substituting (13) and (14) yields $\pi_{2i} = \frac{h(x)}{(N_2-1)h(\bar{x})+h(x)} g \left(\theta \frac{y_2}{y_1} \right) Y_1 \left(1 - x \frac{h'(\bar{x})}{h(x)} \right) < 0$, so firm i will not deviate.

equilibrium¹⁸ is given by

$$r_2 = \frac{g(\theta y^R)}{y^R} f_2'(k_2), \quad (16)$$

$$w_2 = \frac{g(\theta y^R)}{y^R} [f_2(k_2) - k_2 f_2'(k_2)]. \quad (17)$$

3.4.3 Equilibrium

It is immediate that (16) and (17) in the place of (7) and (8) yield that the equilibrium condition is again (11).

3.5 National Accounting in the Presence of Rent-Seeking

Observe that in this economy to produce one unit of good does not imply ownership of it. There are, therefore, three goods in this two-sector economy: the good, the rent-seeking service, and the good in somebody's hands. The price $p_1 = 1 - g$ is the relative price of one unit of the good in units of goods at somebody's hands, and $p_2 = \frac{g}{y^R}$ is the relative price of one unit of the rent-seeking service in units of goods at somebody's hands. Given the equilibrium that we constructed, it follows that

$$\begin{aligned} y_1 &= p_1 y_1 + p_2 y_2 \\ &= l_1(w + rk_1) + l_2(w + rk_2) = p_1(w + rk), \end{aligned}$$

so one can view $p_1 y_1 + p_2 y_2$ as total output of the economy in units of goods at somebody's hands. Moreover, the above equations show that in terms of national accounting, the equilibrium implies that total output can be computed as the sum of the value added in both industries. That is, if rent-seeking is a illegal activity, then total output would be computed as just y_1 (what is produced and then captured by the outlaws). But if it is a legal activity, then its value added is $p_2 y_2 = g y_1$ (that's the "output" of sector 2), and the above equation shows that the usual accounting procedure (both from the value added and from the income perspectives) works. In other words, the static equilibrium takes into account the fact that rent-seeking activities might appear in GDP statistics.¹⁹

¹⁸Recall that for a symmetric equilibrium $\frac{h(\theta Y_{2i})}{\sum_{j=1}^{N_2} h(\theta Y_{2j})} = \frac{1}{N_2}$.

¹⁹For example, consider a small town where there is one firm that offers protection against drug related crimes, on its legal side, and provides drugs, on its illegal side. In effect, this company is a big scam: the fees paid for protection are just a transfer, since there would not be anything to be protected of if the firm didn't exist. But since they are legal fees, they will appear in the national accounts. The equations above show that this is indeed the case in our model.

3.6 Consumers

At a point in time, that is, for given values for k and θ , the static model is solved yielding p , $y_i(p, k)$, and the factor prices r and w . The representative household rents her capital and labor services to the firms. She chooses a consumption plan that solves

$$\begin{aligned} & \max_{c(t)} \int_0^{\infty} e^{-\rho t} u(c(t)) dt \\ \text{s.t. } & \dot{k}(t) = (r(t) - \delta)k(t) + w(t) - c(t), \end{aligned}$$

given $k(0)$, where ρ is the intertemporal discount rate, and δ is the physical depreciation rate.

This is the standard Ramsey problem that yields the following Euler equation

$$\dot{c}(t) = c(t)\gamma(c(t))(r(t) - \rho - \delta), \quad (18)$$

where γ is the intertemporal elasticity of substitution, and $r = (1 - g(\theta y^R(p, k)))f'_1(k_1(p))$ as derived before.

From our discussion in Section 3.5 it follows that

$$r(t)k(t) + w(t) = p_1(t)y_1(t) + p_2(t)y_2(t) = y_1(t),$$

so that the dynamics are represented by the following dynamic system²⁰

$$\left\{ \begin{array}{l} \dot{k} = y_1(p, k) - c - \delta k \\ \dot{c} = c\gamma(c) \left[\underbrace{(1 - g(\theta y^R(p, k))) f'_1(k_1(p))}_r - \rho - \delta \right] \end{array} \right., \quad (19)$$

together with the initial condition for capital, $k(0)$, and the terminal condition $\lim_{t \rightarrow \infty} e^{-\rho t} u'(c(t)) k(t) = 0$, where $p = p(k, \theta)$ is the short-run equilibrium,

The condition for saddle point stability of (19) is that the Jacobian of the linearized system be negative, and this boils down to

$$\left. \frac{k}{r} \frac{dr}{dk} \right|_{\theta} = \left(g \frac{\alpha_g}{1 - \alpha_g} - \frac{1}{\alpha_{2K}} \frac{k_2}{k_1 - k_2} \right) \left. \frac{k}{p} \frac{\partial p}{\partial k} \right|_{\theta} < 0. \quad (20)$$

²⁰The variable t is omitted whenever the understanding is clear.

Appendix C.1 shows that Assumption 3 is a sufficient condition for the inequality above to hold.

3.7 Long Run Equilibrium

The long-run equilibrium is given by a capital stock and a relative price that satisfy the conditions of a steady-state of the dynamic system (19). In other words, the following system of equations must hold in the long-run:

$$\psi_1(p, k) = \frac{g(\theta y^R(p, k))}{y^R(p, k)} \frac{1}{1 - g(\theta y^R(p, k))} - p = 0 \quad (21)$$

$$\psi_2(p, k) = (1 - g(\theta y^R(p, k))) f_1'(k_1(p)) - (\rho + \delta) = 0. \quad (22)$$

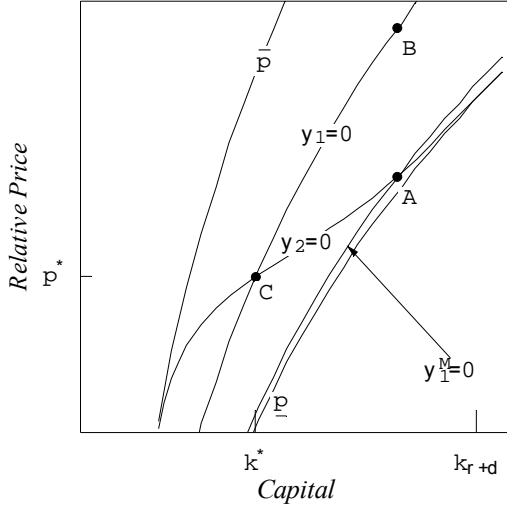


Figure 1: Sector 1 capital intensive

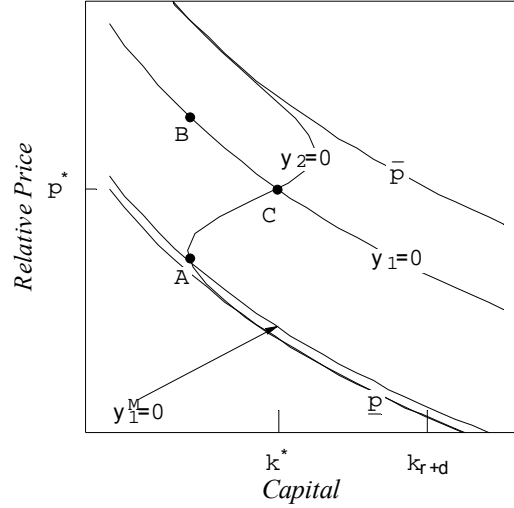


Figure 2: Sector 1 labor intensive

Proposition 2 *The long-run equilibrium exists and is unique.*

Proof. See Appendix B.2. ■

The idea of the proof is shown by Figures 1 and 2 below. They represent the system $\psi_1(p, k) = 0$ and $\psi_2(p, k) = 0$ when the production functions are Cobb-Douglas and the aggregate rent-seeking function is given by (2). (The curve $\psi_1^M = 0$ refers to the monopoly solution of the models. See section 6.) Figure 1 considers sector 1 as capital intensive (the parameter values are $\{\alpha_1, \alpha_2, \alpha_g, \theta, \delta, \rho\} = \{1/3, 1/6, 1/8, 1, \log(1.066), \log(1.03)\}$) and in Figure 2 sector 1 is labor intensive (the parameters are $\{1/6, 1/3, 1/8, 1, \log(1.066), \log(1.03)\}$). As it is clear from the figures, $\psi_1(p, k) = 0$ and $\psi_2(p, k) = 0$ must intersect once, and only once.

4 Properties of the Model

4.1 Comparative Statics

From (11), the effects of k and θ on the static equilibrium p are given by

$$\frac{k}{p} \frac{\partial p}{\partial k} \Big|_{\theta} = - \frac{(1 - \alpha_g) \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p}{1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k} \geq 0 \text{ as } k_1 \geq k_2, \quad (23)$$

$$\frac{\theta}{p} \frac{\partial p}{\partial \theta} \Big|_k = \frac{\alpha_g}{1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k} > 0. \quad (24)$$

In the long-run capital is endogenous and given by (21). The only exogenous variable is θ , the variable that captures the efficiency of the institutional background. When sector 1 is capital intensive the results are intuitive: a deterioration of institutional efficiency generates less capital and output in the long-run. But when the rent-seeking sector is capital intensive, then the reverse result cannot be ruled out. That is, it can be that as the institutional background becomes worse, long-run capital decreases and/or long-run income increases.²¹

4.2 Features of the Dynamics

If the economy is not at its long-run equilibrium, it is at a transition path of capital accumulation. In what follows, it is shown that an economy might be in a dynamic path of capital accumulation with a decreasing level of output.

That is, Appendix C.2 shows that $\frac{dy_1}{dk} \Big|_{\theta} > 0$ if $k_1 \geq k_2$, which means that one can only guarantee that output is increasing along the transition if sector 1 is capital intensive. Analogously, it is also possible to show that $\frac{dy_2}{dk} \Big|_{\theta} > 0$ if $k_1 \leq k_2$. More specifically, Appendix C.2 shows that $\frac{dy^R}{dk} \Big|_{\theta} \leq 0$ as $k_1 \geq k_2$, i.e., that the ratio $\frac{y_2}{y_1}$ is monotone in k : increasing if rent-seeking is capital intensive and decreasing otherwise. Also, the same pattern is followed by the relative value of sector 2's output, $\frac{py_2}{y_1 + py_2}$.

Appendix C.2 also shows that $\frac{dy_1}{dk} \Big|_{\theta}$ is indeterminate when $k_1 < k_2$. Hence there can be a situation where total output of the economy decreases while the capital stock increases. A necessary condition for it is that the rent-seeking sector is capital intensive. Although this configuration is uncommon - the rent-seeking firm produces a service and services are usually labor intensive - it is

²¹Although the indeterminacy is somewhat counter-intuitive, it shows that there is more to the model than just 'bad institutions causing bad economic performance.'

not only a theoretical possibility. Take a very underdeveloped economy (from sub-Saharan Africa for instance). Its productive sector is the agricultural sector. Its unproductive sector is the army and armed bands. It makes sense, then, to consider the rent-seeking sector as the capital intensive sector for this economy. Many sub-Saharan countries have been experiencing negative growth rates and positive investment. One way of explaining it is that investment has been directed mainly to unproductive activities. As an example, Appendix C.2 shows that, for the extreme case that the productive sector only employs labor ($\alpha_{1K} = 0$), the condition $1 > (1 - \alpha_g) \sigma_2$ is sufficient to ensure that $\left. \frac{dy_1}{dk} \right|_{\theta} < 0$, and this is the case as long as $0 \leq \sigma_2 \leq 1$, which is not a strong assumption.

4.3 Scale Effects

The model was solved under the assumption of no technological progress. In one-sector exogenous growth models such assumption can be easily removed, since one only has to re-scale the variables to get a dynamic system with no autonomous part. Such procedure cannot be performed here, unless we assume that both sectors operate under the same technology. If $k_1 \neq k_2$ our model does not deliver a balanced long-run solution. That is, if $k_1 > k_2$ (which seems to be the case for developed economies), then the long-run solution implies that $\frac{y_2}{y_1}$ increases over time (for a given level of institutional efficiency, θ). Likewise, if $k_1 < k_2$ then $\frac{y_2}{y_1}$ decreases over time. The intuition behind this result is simple: with labor-saving technological progress, the labor-intensive industry becomes relatively more productive and attracts relatively more factors of production.

Using the predictions of our model, therefore, we expect an increase in the relative size of the rent-seeking sector in developed economies, for periods of time where the assumption of a given level of institutional efficiency makes sense. An indication of such phenomenon can be inferred from two papers that measure the aggregate burden of crime in US. Becker (1968) estimates that crime accounted for 4% of US's GDP in 1965, while for Anderson (1999) this number rose to 10% in the 90's. Therefore, in about 30 years one part (the crime sector) of the rent-seeking sector increased 150% as a fraction of GDP.²² A corollary of such prediction is that the efficiency of the institutions has to change eventually, otherwise a developed economy would become flooded with rent-seeking activities.²³

²²This fact parallels the increasing share of services in GDP.

²³This provides one reason why institutions change over time. Also, such prediction can be used to test the model: if we have indication that over a long period of time $\frac{y_2}{y_1}$ remained constant, and that $k_1 > k_2$ is a reasonable assumption, then, from the model, institutions must have become more efficient in protecting property rights. For the model to be consistent with these facts, we must also be able to ascertain that such institutional change did take place.

5 Welfare Analysis

The results above show that the dynamics of the model depend of factor intensity. This dependency might be viewed as an indeterminacy. Such is not a concern when the welfare analysis is considered. There is a monotone relation between institutional efficiency and overall welfare in the economy. The worse the institutional background, the lower the welfare enjoyed by the representative consumer. The relevant criterion for evaluating economic performance is welfare, and under such criterion the variable θ does represent the “underlying determinants of economic performance,” as Douglass North would put it.

A deterioration in the institutional set of the economy generates two effects. First, an increase in θ increases p (see (24)) producing an inflow of factors toward the rent-seeking sector and a reduction in the productive sector’s output. This is called Tullock effect. Second, from (5), an increase in θ increases the distortion to capital accumulation (since it reduces $1 - g$). This is called Harberger effect. Under the assumption that initially the economy is in long-run equilibrium, we show that (i) it is possible to disentangle the welfare effect in two components, which are the two above mentioned effects, (ii) the marginal impact of a reduction on institutional efficiency is a reduction in welfare, and (iii) if both sectors operate under the same technology then Harberger effect is zero.

Given that the economy is a representative agent economy, the intertemporal utility is the social welfare function. In order to evaluate the welfare impact of a marginal increase in θ , taken into consideration the transitional dynamics, a technique developed by Judd (1982 and 1987) is employed. Let $W = \int_0^\infty e^{-\rho t} u(c(t)) dt$ be the welfare index. The impact of θ on W at steady state (denoted by an $*$) is:

$$\left. \frac{dW}{d\theta} \right|_* = \int_0^\infty e^{-\rho t} u'(c(t)) \Big|_* \frac{dc(t)}{d\theta} dt = u'(c^*) \int_0^\infty e^{-\rho t} \frac{dc(t)}{d\theta} dt = u'(c^*) C_\theta(\rho),$$

where $X_\theta(\vartheta) \equiv \int_0^\infty e^{-\vartheta t} \frac{dx(t)}{d\theta} dt$ is the Laplace transform of $\frac{dx(t)}{d\theta}$ for any function $x(t)$. Hence, the effect on welfare is given by the Laplace transform ($C_\theta(\rho)$) of $\frac{dc(t)}{d\theta}$ multiplied by marginal utility evaluated at c^* .

Proposition 3 *The impact of θ on W at steady state, $\left. \frac{dW}{d\theta} \right|_*$, is negative. In particular, it is equal to the sum of two components, where the first is always negative and dominates the second. The first is called Tullock effect, and the second is called Harberger effect.*

Proof. Appendix D.1 shows that

$$\rho C_\theta(\rho) = \underbrace{\left. \frac{dy_1}{d\theta} \right|_{k,*}}_{\text{Tullock Effect}} + \underbrace{\text{AF} \left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - \rho - \delta \right) \left. \frac{dk}{d\theta} \right|_*}_{\text{Harberger Effect}}, \quad (25)$$

where μ is the positive eigenvalue associated with the matrix of the linearized dynamic system and

$$\text{AF} = \frac{\mu - \rho}{\mu} \frac{\frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}}{\rho + \delta - \left. \frac{dy_1}{dk} \right|_{\theta,*} + \frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}}.$$

Hence, the impact of θ on welfare is given by the sum of two terms that are identified as Tullock and Harberger effects.

Now, under any configuration Tullock effect is negative (see (24)), Appendix D.2 shows that $0 \leq \text{AF} \leq 1$, and Appendix D.4 shows that $\left| \left. \frac{dy_1}{d\theta} \right|_{k,*} \right| > \left| \left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - \rho - \delta \right) \left. \frac{dk}{d\theta} \right|_* \right|$. It follows that $\left. \frac{dy_1}{d\theta} \right|_{k,*} + \text{AF} \left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - \rho - \delta \right) \left. \frac{dk}{d\theta} \right|_* < 0$, and hence that

$$\left. \frac{dW}{d\theta} \right|_* < 0.$$

■

Therefore, the effect on welfare of a change in the institutional background is unambiguous: welfare is reduced when institutions get less efficient.²⁴

Tullock effect is given by the instantaneous reduction on output, and consequently on consumption, resulting from the deterioration of the institutional set and the corresponding increase in the relative size of the rent-seeking industry.

Harberger effect is the composition of two terms. One is the net marginal impact of capital accumulation on output (net of physical depreciation and of the intertemporal opportunity cost of investment),

$$\left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - \rho - \delta \right) \left. \frac{dk}{d\theta} \right|_*. \quad (26)$$

²⁴Observe an analogy of (25) with the Slutsky equation of consumer theory: a change in θ may be viewed as a change in the price of the consumption good. The total effect is then separated into the substitution effect (the Tullock effect, always of the right sign) and the income effect (the Harberger effect, which has ambiguous sign). What is shown is that the substitution effect always dominates the income effect making the consumption good an “ordinary” good.

The other is the attenuation factor (AF), which translates a change in output due to capital accumulation into a change in welfare taking into consideration the transitional dynamics, The attenuation factor is smaller the smaller is the intertemporal elasticity of substitution $\gamma(c)$. Note that $\lim_{\gamma \rightarrow 0} \text{AF} = 0$ and $\lim_{\gamma \rightarrow \infty} \text{AF} = \lim_{\gamma \rightarrow \infty} \frac{\mu - \rho}{\mu} = 1$.

This is the main qualitative result of the model. It makes a case for improving efficiency of institutions of property rights enforcement based on welfare grounds. Alternatively, it states that the main problem of an unproductive activity is that it employs productive resources that could have been employed socially valued activities. This is the main driving force behind the result that welfare depends positively on institutional efficiency.

Remark 1 *Appendix D.3 shows that, even when the economy is not initially at a steady state position, if the rent-seeking sector is labor intensive, we still have $\frac{dW}{d\theta} < 0$. Better still, the impact of θ on welfare can always be decomposed into Tullock and Harberger effects.*

Remark 2 *Appendix C.2.2 shows that*

$$\left. \frac{dy_1}{dk} \right|_{\theta,*} - \rho - \delta \geq 0 \text{ as } k_1 \geq k_2,$$

so, from (26), Harberger effect is zero if $k_1 = k_2$.²⁵

This last result can be interpreted as follows: with rent-seeking, k is not only a good, it can also be a bad (when employed in sector 2); as k increases, the economy gains because k_1 increases, and loses because k_2 also increases. Those two effects cancel out when technologies are equal in both sectors. One can argue, then, that Harberger effect on welfare is of second order. Not only it is always dominated by Tullock effect, it is only non zero when technologies are not the same in both sectors.

6 Monopoly

The model presented in this paper assumes that the rent-seeking sector is a competitive industry. The free entry condition and the assumption of an optimum plant size for each firm in sector 2 guarantee that rents are fully dissipated in equilibrium. It seems reasonable to assume that profit

²⁵ Under this configuration ($k_1 = k_2$) it follows from the short-run equilibrium condition that the share of workers in the rent-seeking sector, l_2 , is equal to the share of output extracted by the rent-seeking sector, g . Consequently, the marginal impact of capital on output, $l_1 f'(k)$, is equal to the market interest rate, $(1 - g) f'(k)$, which implies that the social value of capital is equal to the private one.

opportunities will be taken up by someone in a society, so the assumption of a large number of rent-seekers has its appeal. It is plain that different forms of market organization could be considered. We began with the competitive paradigm because we view it as the relevant scenario for a market economy, especially in the long-run. In this section, the model with just one firm operating in sector 2 is considered.²⁶ With such a model, one can compare the results of the previous model and also, as was argued in the Introduction, compare the effects of rent-seeking in open societies with rent-seeking in more closed societies.

It is possible to imagine a situation in which there is a central organization that gives right to a unique firm to practice rent-seeking.²⁷ Assume that this central organization does exist and that it is able to enforce this right.²⁸ The monopolist uses its market power to make positive profits and generates less rent-seeking than a competitive industry does.²⁹

The monopolist problem is to solve

$$\max_{K_2, L_2} g\left(\theta \frac{L_2 f_2(k_2)}{Y_1}\right) Y_1 - r_2 K_2 - w_2 L_2,$$

which yields

$$\begin{aligned} r_2 &= \theta g'(\theta y^R) f_2'(k_2) \\ w_2 &= \theta g'(\theta y^R) [f_2(k_2) - k_2 f_2'(k_2)]. \end{aligned}$$

These two equations replace (16) and (17) for the competitive economy.

The argument to characterize the static equilibrium is the same as before. The marginal rate of transformation determined by y^R is now $\frac{\theta g'(\theta y^R)}{1-g(\theta y^R)}$ and, for each given MRT, the relative price p^M determines the relative supply y^R , so that the equilibrium is a fixed point as before³⁰

$$H^M(p^M) \equiv \frac{\theta g'(\theta y^R(p^M))}{1-g(\theta y^R(p^M))} = p^M. \quad (27)$$

With such equilibrium in hands, we can state:

²⁶By doing that we analyze the two polar cases of market organization. Other market structures are likely to generate conclusions lying somewhere in between the two extreme cases.

²⁷One can imagine that this monopolist is the communist party. See Section ??.

²⁸We can think, instead, that there are many firms which are working as a cartel, maximizing jointly their profit. The key hypothesis here is limited entry in the rent-seeking sector.

²⁹In order to close the model in general equilibrium we can think that each individual in the society is the owner of an equal share of the rent-seeking firm, such that the profit is redistributed back to the household in a lump-sum fashion.

³⁰Clearly, the interpretation in terms of factor mobility is still valid.

Proposition 4 *In the short run (for given levels of θ and k), output per capita is larger for an economy with a monopolist rent-seeker than for an economy with a competitive rent-seeking sector.*

Proof. Strict concavity of g implies that, for a given level of θy^R , $g'(\theta y^R) < \frac{g(\theta y^R)}{\theta y^R}$, or that $H^M(p) < H(p)$. Under Assumption 4 it is straightforward to show that $\left. \frac{\partial H^M}{\partial p} \right|_k < 0$. Given that $H^M(p^M)|_{\psi_1=0} < 0$ it follows that $H^M(p^M) - p^M = 0$ lies somewhere in the middle of the stripe connecting $\psi_1 = 0$ and $\underline{p}(k)$ (see Figures 1 and 2, where is it depicted as the curve $\psi_1^M = 0$). Consequently, a fixed point p^M of H^M must be smaller than the equilibrium price of the competitive model: $p^M < p$. This implies that

$$y_1(p^M(\theta, k), k) > y_1(p(\theta, k), k).$$

■

Hence, for given values of k and θ , a monopoly in sector 2 is better for sector 1. Less output gets confiscated by sector 2. Competition improves welfare as long as it is employed in productive sectors of an economy. In an unproductive sector, given that competitors do not internalize the reduction in aggregate output due to their action, competition means too much production of transfer services, and consequently too much taken away from the productive sector and too much productive factors allocated to unproductive activities.

The long-run part is as before. The same intertemporal decision leads to a dynamic system like (19) with p^M instead of p . Consequently, the long-run equilibrium is described by the crossing of $\psi_2 = 0$ and $\psi_1^M = 0$ in Figures 1 and 2. Given that $\psi_2 = 0$ crosses $\psi_1 = 0$ and intersects $\underline{p}(k)$ at $k_{\rho+\delta}$, existence follows from the same argument as before. We can also state:

Proposition 5 *In the long run (for a given value of θ), output per capita is larger for an economy with a monopolist rent-seeker than for an economy with a competitive rent-seeking sector.*

Proof. Appendix E shows taking into consideration the long-run endogenous capital adjustment we have

$$y_1^M(\theta) > y_1(\theta).$$

■

This result is not immediate. The monopoly solution also delivers ambiguous results in the long-run comparative statics with respect to θ , so one could expect that the ambiguity would carry on to the comparison between y_1^M and y_1 . It turns out that this is not the case, and that the

short-run result holds in the long-run as well. The relatively smaller monopoly’s output is a benefit for the economy as a whole, as overall output (GDP) is larger.

At this point it is possible to analyze the transition from centralized political or economic systems to more open systems.³¹ Consider an economy in long-run equilibrium with monopoly in sector 2 (point A at Figure 1 or 2). Following a change of system, the economy jumps to B, where it begins a dynamic path toward C, the long-run equilibrium for competitive rent-seeking and the same θ .³² The jump from A to B represents an unambiguous decrease in output. Given that output in C is lower than output in A and that along the path from B toward C output is lower than output in A it follows that welfare is reduced after the introduction of competitive rent-seeking.

7 Calibration

In this section the model is solved with particular functional forms and observed data is used to compute the implied values for the relevant parameters. The idea is to show how well the model fits the data and also to illustrate the difference between the competitive and monopoly solutions. We assume that the two sectors operate under the same technology (Cobb-Douglas with parameter α) and that the aggregate rent-seeking technology is given by the “Cobb-Douglas-like” form (equation (2)). With equal technologies it follows that $y^R = \frac{l_2 f(k)}{l_1 f(k)} \equiv l^R$. The long-run equilibrium condition (equation (21)) becomes $\frac{\alpha k^{\alpha-1}}{1+(\theta l^R)^{\alpha g}} = \rho + \delta$, and the short-run conditions become $\theta^{\alpha g} (l^R)^{\alpha g-1} = 1$ and $\theta \frac{\alpha g (\theta l^R)^{\alpha g-1}}{1+(\theta l^R)^{\alpha g}} = 1$, for the competitive economy and monopoly respectively (equations (22) and (27))

At this stage, two parameters are not observable in principle, θ and αg . Instead of calibrating the first, we use a proxy for it. Given that it is meant to be a measure of the quality of the institutions of an economy, there is an observed variable that measures such a parameter. It is the variable SI (social infrastructure) created by Hall and Jones (1999), which is an index of institutional efficiency ranging from 0 to 1, 1 being the highest degree of institutional efficiency. We assume that

$$\theta = B \frac{1 - \text{SI}}{\text{SI}}, \quad (28)$$

where B has to be calibrated. In other words the observable counterpart for θ is an increasing mapping on $\mathbb{R}^{[0,1]}$ of the observable SI with an adjustable Jacobian given by B .³³ So now the set

³¹See further elaboration of this point in Section 8.

³²Note that A might lie to the right of point C when sector 2 is capital intensive.

³³If such increasing mapping is also required to be convex, differentiable, and to satisfy Inada-like conditions, then our formulation is without loss of generality: any such transformation can be approximated by our transformation

$\{B, \alpha_g\}$ of parameters has to be calibrated, and for that two observables are needed.

The first observable comes from Anderson (1999). He reports that the aggregate burden of crime in the US is around 10% of GDP. Under the extreme assumption that rent-seeking in the US is mainly crime (US is indeed one of the most efficient economies in the world, so one might think of that assumption as a normalizing assumption that sets US's institutional inefficiency outside crime to zero) we consider $l^{R,US} = \frac{0.1}{0.9} = \frac{1}{9}$, or $\frac{l_1^{US}}{l_1^{NN}} = \frac{9}{10}$, where the superscript NN stands for 'neoclassical nirvana,' which is the situation with $\theta = 0$. According to the data set of Hall and Jones (1999), $SI_{US} = 0.973$. Given that the long-run solution of the competitive model is $\frac{y^{US}}{y^{NN}} = \left(1 + \theta \frac{\alpha_g}{1-\alpha_g}\right)^{\frac{1}{\alpha-1}}$, and assuming $\alpha = \frac{1}{3}$, we get

$$B = \frac{0.973}{1 - 0.973} \left[(0.9)^{-\frac{2}{3}} - 1 \right]^{\frac{1-\alpha_g}{\alpha_g}}. \quad (29)$$

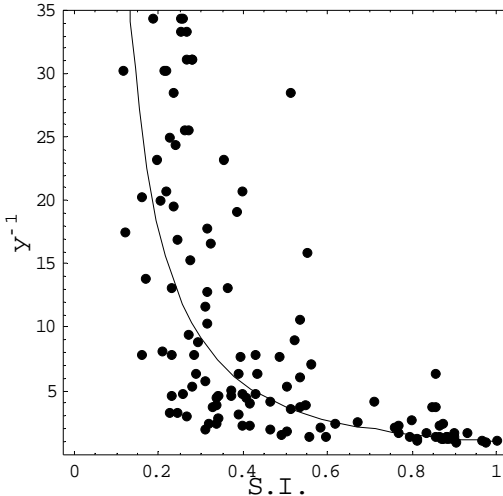


Figure 3

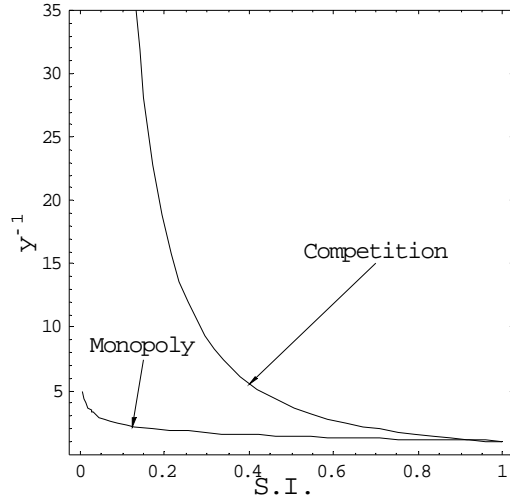


Figure 4

A second observable is needed to match the curvature parameter α_g . Hall and Jones (1999) estimated the equation $\log y_i = \beta_0 + \beta_1 SI_i + \epsilon_i$. Their estimate for β_1 is 5.14. Assuming that this result describes well the relation between output and institutional efficiency, we consider α_g as the solution of the following problem:

$$\min_{\alpha_g, \zeta} \int_0^1 \left\{ \zeta + 5.14 SI - \log \left[1 + \left(B \frac{1 - SI}{SI} \right)^{\frac{\alpha_g}{1-\alpha_g}} \right]^{\frac{1}{\alpha-1}} \right\}^2 d(SI),$$

(28) by an appropriate choice of the Jacobian B .

where B is given by (29).³⁴ That is, we consider α_g that minimizes the distance between $\zeta + 5.14\text{SI}$ and the logarithm of the income when θ is given by $B\frac{1-\text{SI}}{\text{SI}}$. The solution is $\alpha_g = 0.506$, which is well inside the stability region of $\alpha_g < 1 - \alpha = \frac{2}{3}$.

Figure 3 below is the scatter diagram of the Hall and Jones data set for SI and per capita income,³⁵ together with the long-run solution of the model with $\alpha_g = 0.506$ and $B = 1.391$. It is apparent that the pattern of the data is reproduced by the model. There is a negative relation between institutional inefficiency and output as expected. The fact that the fit is a good one shows that the model can potentially explain the data. More importantly, the introduction of unproductive activities improves the predictive power of the neoclassical growth model.

Figure 4 displays the prediction of the model under the two configurations: competitive rent-seeking and monopoly. It is apparent that the competitive formulation generates much more rent-seeking for each level of θ (or SI). This is in line with the interpretation that the monopoly solution is better for the rest of the economy.

8 Applications

The results presented above show that the model can be used as a framework to study unproductive activities from a macroeconomic perspective. It provides a simple structure that generates the stylized facts about rent-seeking and growth. Below we show that such framework can deliver more than those stylized facts: we present three applications showing that the model can be used to analyze some interesting issues in growth and development.

8.1 Endogenous TFP

The share of productive factors allocated in sector 1 can be viewed as an endogenous part of the TFP. The output of an economy increases (decreases) as productive factors move from sector 2 (1) to sector 1 (2), and this happens for a given level of productive factors. So this is not accounted as a change in output due to a change in productive factors, but due to a change in the productivity of the existing factors, i.e., a change in TFP. More formally, consider first the case with $k_1 = k_2$. Then $y_1 = l_1 f(k) = (1 - l_2) f(k)$, and the term $1 - l_2$ can be viewed as (part of) the TFP. The bigger the rent-seeking sector, the less productive the economy. That is, for a given k , an increase

³⁴The constant ζ is an unimportant level parameter. Notice that we could have calibrated α_g directly from the data but we decided to use Hall and Jones's regression because they controlled for endogeneity of SI. See further evidence in Acemoglu, Johnson and Robinson (2001), who also argue that the instrument used by Hall and Jones is appropriate.

³⁵The values of per capita income are net of mining activities as a way of controlling for natural resources and refers to the year of 1988. Further data information can be found in their paper.

in l_2 represents a decrease in TFP, since less output is produced by the same level of productive factors, k . The same intuition is also valid for generic values of k_1 and k_2 .

In other words, let us assume that our model describes well two economies that are identical in every aspect but differ in the parameter θ . Then an observer looking at these economies from the viewpoint of an one-sector aggregate model would conclude that the economy with the smaller θ is the economy with higher TFP, although both economies operate under the same technology by assumption. In this sense, TFP (or part of it) is endogenously determined by the institutional efficiency.

Moreover, there is also a dynamic issue in this endogenous TFP. An once-and-for-all change in institutional efficiency is given by a discrete jump in θ .³⁶ This generates an immediate reallocation of factors between the sectors, and so an immediate change in TFP. But the economy enters in a transitory dynamic path towards its new steady state, and along this path further reallocations of factors take place. That is, along this path the share of the labor force allocated in the productive sector keeps changing and this is observationally equivalent to a continuous change of the TFP if an one-sector economy perspective is considered. Such dynamic behavior is to be contrasted with the usual exercises in the literature of considering once and for all changes in TFP itself: the resulting transitory dynamics of capital accumulation does not include an associated transitory dynamics of TFP. Also, there is some evidence that TFP is indeed not constant. A simple look at the Summers and Heston data set reveals that several countries, like Venezuela, endured a process of reduction in TFP for the period of 1960 to 1990. Other countries, like Ireland in the 90's, endured the opposite process. The model provides an immediate rationale for such facts, as one can easily argue that Venezuela and Ireland witnessed changes in institutional efficiency prior to (or during) the period in question. The issue of endogeneity of TFP is of great interest (see Prescott (1998)) and ought to be studied further.

8.2 Transition Economies

The monopoly result presented in Section 6 can be used to describe some dynamics that ensued from three different historical events of the second half of last century: the end of European colonization in the 60's and 70's in many African countries, the end of political regimes based on military dictatorship in many Latin America countries in the 80's, and finally the 'fall of the wall' leading to the end of the communist regimes in east Europe in the late 80's and early 90's. These three recent episodes of the world history share one fundamental characteristic: there is a transition from a centralized political (economic) organization toward a more decentralized system. And

³⁶Rodrik (1999) suggested one possible mechanism that can produce such change.

such transitions were all accompanied by a period of economic recession. One rationale for that is provided by our monopoly result. That is, assuming that monopoly in rent-seeking takes place in either a colony (the European imperial power being the monopolist),³⁷ or in a military dictatorship (the army being the monopolist), or a centralized economy (the communist party being the monopolist), a given level of institutional efficiency is associated with a better economic performance in the more centralized system. Also, the transition to a more open system of organizing either the politics or the economy means a lifting of the barriers to entry in the rent-seeking sector, so the economy is bound to experience a recession as productive resources are directed to unproductive activities.

8.3 Foreign Aid and Rent-Seeking

Consider now the issue of foreign aid in the presence of rent-seeking. There is a concern that aid, if the recipient economy does not have a good institutional set, is a waste of resources: it ends up as consumption, without any effect on the productive capacity of the economy (Burnside and Dollar (2000)). The model shows that things might be even worse when rent-seeking is considered: the aid generates an increase in rent-seeking activities, so that the society would be better off if the aid was given directly as consumption goods to the households. To see this consider the aid as a permanent flow, A , of resources per capita, so that per capita income becomes $y_1 + A$. The short-run equilibrium condition (11) is still valid, now with $y^R = \frac{y_2}{y_1 + A}$. Consequently, in the short-run an increase in A induces factors of production to move into the rent-seeking sector. Given the increase in the “pie,” there are more resources to be stolen, and hence the transfer efforts increase. In the long-run an increase in aid leads to an increase in y_2 and a *decrease* in y_1 . Hence there is an unambiguous increase in rent-seeking activities generated by the foreign aid.

9 Concluding Remarks

As mentioned above, we have taken an aggregate perspective to modelling rent-seeking in an economy. Our goal was to provide a simple formalization to help understanding the impact of unproductive activities in economic growth. It is our understanding that such a formalization has not yet been provided in the literature of economic growth. We have not provided new insights on the causes of unproductive activities. Rather, we summarized such causes in a reduced form parameter θ representing the efficiency of the institutional set in preventing such activities. We also used a reduced form “aggregate rent-seeking” function to describe how unproductive activities capture

³⁷Lucas (1990) considered the case in which the European power is the monopolist in the capital market.

rents from productive activities. Hence our starting point was that unproductive activities exist and ought to be considered in models of economic growth. Such approach is analogous to the use of the cash-in-advance constraint in monetary models: money exists and ought to be considered in macroeconomic models. Instead of explaining why money exists, such models focus on the implications of such existence. Likewise, our focus is on the implications for economic development of the existence of unproductive activities.

We also would like to point out that unproductive activities cannot be viewed as the sole factor missing in the standard models of economic growth. A complete picture ought to include every factor that affects the accumulation as well as the quality of productive factors. Unproductive activities (as modelled here) affect only the accumulation of capital, and are mute with respect to accumulation of human capital and technological change. A more general description of the impact of unproductive activities should include their effects on the accumulation of human capital, on technological change and on the labor supply decision, among other factors. Such effects are likely to increase the power of our parameter θ in explaining the observed differences in per capita income among countries. Nevertheless, our calibration results show that unproductive activities are an important element in helping explaining long-run performance of economies, even without considering the effects mentioned above.

A The Static Model: Existence and Comparative Statics Properties

A.1 The Two Sector Model of Production

The following equations describe the 2×2 static model:

$$y_i = \frac{L_i}{L} f_i \left(\frac{K_i}{L_i} \right) = l_i f_i(k_i) \quad i = 1, 2, \quad (30)$$

$$l_1 + l_2 = 1, \quad (31)$$

$$k_1 l_1 + k_2 l_2 = k, \quad (32)$$

$$w = p_i (f_i - k_i f_i'), \quad (33)$$

$$r = p_i f_i'', \quad (34)$$

where

$$f_i(0) = 0, f_i'(k_i) > 0, \lim_{k_i \rightarrow 0} f_i'(k_i) = \infty, \lim_{k_i \rightarrow \infty} f_i'(k_i) = 0, f_i''(k_i) < 0.$$

From (33) and (34) we get

$$\omega \equiv \frac{w}{r} = \frac{f_i}{f'_i} - k_i, \quad (35)$$

or

$$k_i = k_i(\omega), \quad \frac{dk_i}{d\omega} = -\frac{(f'_i)^2}{f_i f''_i} > 0. \quad (36)$$

The relative price is the ratio of average costs:

$$p \equiv \frac{p_2}{p_1} = \frac{\frac{wL_2+rK_2}{L_2f_2}}{\frac{wL_1+rK_1}{L_1f_1}} = \frac{\omega + k_2 \frac{f_1}{f_2}}{\omega + k_1 \frac{f_1}{f_2}},$$

which is solved as

$$\omega = \omega(p), \quad \frac{p \, d\omega}{\omega \, dp} = \frac{(\omega + k_1)(\omega + k_2)}{\omega(k_1 - k_2)} \geq 0 \text{ as } k_1 \geq k_2. \quad (37)$$

Solving (31) and (32) for l_i , after substituting into (30) we get the supply functions:

$$y_1(p, k) = l_1 f_1(k_1) = \frac{k_2(\omega(p)) - k}{k_2(\omega(p)) - k_1(\omega(p))} f_1(k_1(\omega(p))), \quad (38)$$

$$y_2(p, k) = l_2 f_2(k_2) = \frac{k - k_1(\omega(p))}{k_2(\omega(p)) - k_1(\omega(p))} f_2(k_2(\omega(p))). \quad (39)$$

Usually we will write simply $k_i(\omega(p)) = k_i(p)$.

Finally, for a given factor endowment, there is a price $\bar{p}(k)$ such that the economy is specialized in the production of the rent-seeking service if $p \geq \bar{p}(k)$, and there is a price $\underline{p}(k)$ such that the economy is specialized in the production of the first sector good if $p \leq \underline{p}(k)$. Note that $\bar{p}'(k) \geq 0$ and $\underline{p}'(k) \geq 0$ as $k_1 \geq k_2$.³⁸

³⁸See Kemp (1969), chapter 1.

A.2 Comparative Statics

The following notation is employed from now on:

$$\alpha_{iK} \equiv 1 - \alpha_{iL} = k_i \frac{f'_i}{f_i}, \quad (40)$$

$$\sigma_i \equiv \frac{\omega}{k_i} \frac{dk_i}{d\omega} = \frac{\alpha_{iL}}{\alpha_{iK}} \frac{dk_i}{d\omega} = -\frac{\alpha_{iL}}{\alpha_{iK}} \frac{(f'_i)^2}{f_i f''_i}. \quad (41)$$

An important consequence of (35) is that

$$\frac{k_1}{k_2} = \frac{1 - \frac{f_1}{k_1 f'_1}}{1 - \frac{f_2}{k_2 f'_2}} = \frac{\alpha_{2L}}{\alpha_{1L}} \frac{\alpha_{1K}}{\alpha_{2K}} = \frac{\alpha_{1K}}{1 - \alpha_{1K}} \frac{1 - \alpha_{2K}}{\alpha_{2K}} \geq 1 \text{ whether } \alpha_{1K} \geq \alpha_{2K}. \quad (42)$$

The comparative statics for (38) and (39) are:

$$\left. \frac{k}{y_1} \frac{\partial y_1}{\partial k} \right|_p = \frac{k}{k - k_2} = \frac{1}{l_1} \frac{k}{k_1 - k_2}, \quad (43)$$

$$\left. \frac{k}{y_2} \frac{\partial y_2}{\partial k} \right|_p = \frac{k}{k - k_1} = -\frac{1}{l_2} \frac{k}{k_1 - k_2} = -\frac{1}{l^R} \left. \frac{k}{y_1} \frac{\partial y_1}{\partial k} \right|_p, \quad (44)$$

and

$$\left. \frac{p}{y_1} \frac{\partial y_1}{\partial p} \right|_k = -\frac{\alpha_{1L}}{(\alpha_{1K} - \alpha_{2K})^2} (\sigma_2 \alpha_{2K} l^R + \sigma_1 \alpha_{1K}), \quad (45)$$

$$\left. \frac{p}{y_2} \frac{\partial y_2}{\partial p} \right|_k = \frac{\alpha_{2L}}{(\alpha_{1K} - \alpha_{2K})^2} \left(\sigma_1 \alpha_{1K} \frac{1}{l^R} + \sigma_2 \alpha_{2K} \right) = -\frac{1}{l^R} \frac{\alpha_{2L}}{\alpha_{1L}} \left. \frac{p}{y_1} \frac{\partial y_1}{\partial p} \right|_k, \quad (46)$$

where $l^R \equiv \frac{l_2}{l_1}$. In deriving this last two equations we employed (32), (35)-(37), (40)-(42). Two important results that follow from (43) and (44) are:

$$\left. \frac{1 - g}{r} \frac{\partial y_1}{\partial k} \right|_p = \frac{\alpha_{2L}}{\alpha_{1K} - \alpha_{2K}}, \quad (47)$$

$$\left. \frac{1 - g}{r} \frac{\partial y_2}{\partial k} \right|_p = -\frac{y^R}{l^R} \frac{\alpha_{2L}}{\alpha_{1K} - \alpha_{2K}} = -\frac{1}{p} \frac{\alpha_{1L}}{\alpha_{1K} - \alpha_{2K}}, \quad (48)$$

where in the last equation we substitute (11) and (24).

There are two results from the two-sectors general equilibrium model of production that we can

use:

$$\left. \frac{\partial y_1}{\partial p} \right|_k + p \left. \frac{\partial y_2}{\partial p} \right|_k = 0, \quad (49)$$

$$\left. \frac{\partial}{\partial k} (y_1 + py_2) \right|_p = \left. \frac{\partial y_1}{\partial k} \right|_p + p \left. \frac{\partial y_2}{\partial k} \right|_p = \frac{r}{1-g}. \quad (50)$$

(The reader can try to show directly these two results. (49) follows from (45) and (46) recalling the short-run equilibrium condition. (50) follows from (47) and (48).)

B Existence and Uniqueness of Equilibria

B.1 Short Run (Proposition 1)

Let $H : [\underline{p}, \bar{p}] \rightarrow R_+$ be the mapping defined by $H(p) \equiv H(p, k, \theta)$ for given (k, θ) , so that the short-run equilibrium $p(k, \theta)$ is given by a fixed point of H . Taking derivatives and rearranging:

$$\frac{p}{H(p)} \frac{dH(p)}{dp} = -(1 - \alpha_g) \frac{p}{y^R(p)} \frac{dy^R(p)}{dp} < 0,$$

and the inequality follows from $\alpha_g < 1$. From Assumption 1 we have $\lim_{p \rightarrow \underline{p}^+} H(p) = \lim_{y^R \rightarrow 0} \frac{g(\theta y^R)}{y^R} \geq \lim_{y^R \rightarrow 0} g'(\theta y^R) = \infty$. From Assumption 2, integrating α_g gives $g(x) \leq \frac{x^{\bar{\alpha}_g}}{1+x^{\bar{\alpha}_g}}$. Hence $\lim_{x \rightarrow \infty} x(1-g(x)) \geq \lim_{x \rightarrow \infty} \frac{x}{1+x^{\bar{\alpha}_g}} = \infty$, which ensures that $\lim_{p \rightarrow \bar{p}} H(p) = \lim_{y^R \rightarrow \infty} \frac{g(\theta y^R)}{y^R} \frac{1}{1-g(\theta y^R)} = 0$.

B.2 Long Run (Proposition 2)

Let $f'_1(k_{\rho+\delta}) = \rho + \delta$ and $\psi_1(p_{\rho+\delta}, k_{\rho+\delta}) = 0$. Given that on $\underline{p}(k)$ the economy is specialized in the production of the productive good,

$$r|_{(\underline{p}(k_{\rho+\delta}), k_{\rho+\delta})} = \rho + \delta \Rightarrow (\underline{p}(k_{\rho+\delta}), k_{\rho+\delta}) \in [\psi_2 = 0].$$

Additionally, we know that $r = (1-g) f'_1|_{(p_{\rho+\delta}, k_{\rho+\delta}) \in \psi_1} < \rho + \delta$. In order to show that there is a point on $\psi_1 = 0$ such that $r = \rho + \delta$ we show that $\lim_{p \rightarrow 0} r = \infty$. Given Assumption 2, write $\alpha_g(x) \leq \bar{\alpha}_g - \varepsilon$, for some $\varepsilon > 0$. From (20)

$$\frac{p}{r} \frac{dr}{dp} = g \frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1L}}{1-\alpha_{2K}-\alpha_{1L}} \leq \frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1L}}{1-\alpha_{1L}} \leq -\beta,$$

where $\beta \equiv \frac{\varepsilon}{(1-\alpha_{1L})(1-\alpha_{1L}+\varepsilon)} > 0$. Let $(p_0, k_0) \in \psi_1$, and let $r_0 = (1-g) f'_1|_{(p_0, k_0)}$. Integrating the last inequality yields

$$\frac{r}{r_0} \geq \left(\frac{p}{p_0}\right)^{-\beta},$$

for any $p \leq p_0$. This shows existence. For uniqueness, notice that since $0 > \frac{dr}{dk}|_{\theta} = \frac{\partial r}{\partial k}|_{p, \theta} + \frac{\partial r}{\partial p}|_{k, \theta} \frac{dp}{dk}|_{\psi_1, *}$, and $\frac{dp}{dk}|_{\psi_2, *} = -\frac{\frac{\partial r}{\partial k}|_{p, \theta}}{\frac{\partial r}{\partial p}|_{k, \theta}}$, we have

$$\frac{dp}{dk}|_{\psi_2, *} = \frac{\frac{\partial r}{\partial k}|_{p, \theta}}{\frac{\partial r}{\partial k}|_{p, \theta} - \frac{dr}{dk}|_{\theta}} \frac{dp}{dk}|_{\psi_1, *} \leq \frac{dp}{dk}|_{\psi_1, *} \text{ as } k_1 \geq k_2,$$

so the curves intersect only once.³⁹

C Dynamics and Long-Run Equilibrium

C.1 Stability

The saddle point stability for the dynamic system (19) requires

$$\frac{dr}{dk}|_{\theta} = \frac{d}{dk} (1 - g(\theta y^R(p, k))) f'_1(k_1(p)) \Big|_{\theta} < 0.$$

But

$$\frac{k}{y^R} \frac{dy^R}{dk} \Big|_{\theta} = \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p + \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k \frac{k}{p} \frac{\partial p}{\partial k} \Big|_{\theta} = -\frac{1}{1-\alpha_g} \frac{k}{p} \frac{\partial p}{\partial k} \Big|_{\theta}, \quad (51)$$

where in the last equality (23) was used. Consequently, using the definition of α_g , (40) and (41), we get

$$\frac{k}{r} \frac{dr}{dk} \Big|_{\theta} = \left(g \frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1L}}{\sigma_1} \frac{p}{k_1} \frac{dk_1}{dp} \right) \frac{k}{p} \frac{\partial p}{\partial k} \Big|_{\theta}.$$

After substituting (35)-(37) we get

$$\frac{k}{r} \frac{dr}{dk} \Big|_{\theta} = \left(g \frac{\alpha_g}{1-\alpha_g} - \frac{1}{\alpha_{2K}} \frac{k_2}{k_1 - k_2} \right) \frac{k}{p} \frac{\partial p}{\partial k} \Big|_{\theta} = \left(g \frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1L}}{1-\alpha_{2K}-\alpha_{1L}} \right) \frac{k}{p} \frac{\partial p}{\partial k} \Big|_{\theta}, \quad (52)$$

³⁹Recall that $\frac{dp}{dk}|_{\psi_1, *} \geq 0$ as $k_1 \geq k_2$.

where in this last equality we used (42). A sufficient condition for $\frac{k}{r} \frac{dr}{dk} \Big|_{\theta} < 0$ when $\alpha_{1K} > \alpha_{2K}$ is Assumption 3 ($\alpha_{1L} > \alpha_g$), because

$$\frac{\alpha_{1L}}{1 - \alpha_{2K} - \alpha_{1L}} \geq \frac{\alpha_{1L}}{1 - \alpha_{1L}} \geq \frac{\alpha_g}{1 - \alpha_g}.$$

C.2 Features of the Dynamics

C.2.1 Behavior of y_1 and y_2

After substituting (23) we get

$$\begin{aligned} \frac{dy_1}{dk} \Big|_{\theta} &= \frac{\partial y_1}{\partial k} \Big|_p + \frac{\partial y_1}{\partial p} \Big|_k \frac{\partial p}{\partial k} \Big|_{\theta} = \frac{\partial y_1}{\partial k} \Big|_p - \frac{\partial y_1}{\partial p} \Big|_k \frac{p}{k} \frac{(1 - \alpha_g) \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p}{1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k} \\ &= \frac{1}{\Delta} \left\{ \frac{\partial y_1}{\partial k} \Big|_p \left[1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k \right] - (1 - \alpha_g) \frac{\partial y_1}{\partial p} \Big|_k \frac{p}{y^R} \frac{\partial y^R}{\partial k} \Big|_p \right\}, \end{aligned}$$

where

$$\Delta \equiv 1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k.$$

After employing (49) and (50) we get:

$$\frac{dy_1}{dk} \Big|_{\theta} = r \frac{\frac{1}{r} \frac{\partial y_1}{\partial k} \Big|_p + \frac{1 - \alpha_g}{1 - g} \frac{p}{y_2} \frac{\partial y_2}{\partial p} \Big|_k}{1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k} > 0 \quad (53)$$

if $k_1 \geq k_2$.

Analogously it is possible to show that

$$\frac{dy_2}{dk} \Big|_{\theta} = r \frac{\frac{1}{r} \frac{\partial y_2}{\partial k} \Big|_p - \frac{1 - \alpha_g}{1 - g} \frac{1}{y_1} \frac{\partial y_1}{\partial p} \Big|_k}{1 + (1 - \alpha_g) \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k} > 0 \quad (54)$$

if $k_2 \geq k_1$.

Finally, it follows from (53) that

$$\frac{dy_1}{dk} \Big|_{\theta} \geq 0 \text{ as } \frac{1}{r} \frac{\partial y_1}{\partial k} \Big|_p + (1 - \alpha_g) \frac{p}{y_2} \frac{\partial y_2}{\partial p} \Big|_k \frac{1}{1 - g} \geq 0.$$

After substituting (43) and (46) in this last expression we get

$$\left. \frac{dy_1}{dk} \right|_{\theta} \geq 0 \text{ as } -1 + \frac{1 - \alpha_g}{\alpha_{2K} - \alpha_{1K}} \left(\sigma_1 \alpha_{1K} \frac{1}{l^R} + \sigma_2 \alpha_{2K} \right) \geq 0.$$

C.2.2 Social Value of Capital

It follows from the short-run equilibrium condition, expression (11) and from (49) that

$$(1 - g) \left. \frac{p}{y_1} \frac{\partial y_1}{\partial p} \right|_k + g \left. \frac{p}{y_2} \frac{\partial y_2}{\partial p} \right|_k = 0.$$

With the help of this last equation and (53) one can show that

$$\frac{k}{y_1} \left(\left. \frac{dy_1}{dk} \right|_{\theta} - r \right) = -\frac{g}{\Delta} \left. \frac{k}{y^R} \frac{\partial y^R}{\partial k} \right|_p \geq 0 \text{ as } k_1 \geq k_2. \quad (55)$$

C.2.3 Behavior of y^R

Note that a consequence of these last two results is that, after employing (49)

$$\begin{aligned} \left. \frac{k}{y^R} \frac{dy^R}{dk} \right|_{\theta} &= \left. \frac{k}{y_2} \frac{dy_2}{dk} \right|_{\theta} - \left. \frac{k}{y_1} \frac{dy_1}{dk} \right|_{\theta} = \frac{\left. \frac{k}{y_2} \frac{\partial y_2}{\partial k} \right|_p - \left. \frac{k}{y_1} \frac{\partial y_1}{\partial k} \right|_p}{1 + (1 - \alpha_g) \left. \frac{p}{y^R} \frac{\partial y^R}{\partial p} \right|_k} \\ &= \frac{1}{\Delta} \left. \frac{k}{y^R} \frac{\partial y^R}{\partial k} \right|_p \leq 0 \text{ as } k_1 \geq k_2. \end{aligned}$$

C.3 Long-Run Capital Stock

Given that in the long-run the interest rate is fixed at $\rho + \delta$, the effect of θ on k is given by $\left. \frac{\theta}{k} \frac{dk}{d\theta} \right|_k = -\frac{\left. \frac{\theta}{r} \frac{dr}{d\theta} \right|_k}{\left. \frac{k}{r} \frac{dr}{dk} \right|_{\theta}}$. It follows from the stability, condition (20), that

$$\left. \frac{dk}{d\theta} \right|_k \geq 0 \text{ as } \left. \frac{dr}{d\theta} \right|_k \geq 0.$$

Following the same steps taken in Appendix B.1, recalling that (24) implies that

$$g\alpha_g \left(1 + \left. \frac{p}{y^R} \frac{\partial y^R}{\partial p} \right|_k \frac{\left. \theta}{p} \frac{\partial p}{\partial \theta} \right|_k \right) = g \left(1 + \left. \frac{p}{y^R} \frac{\partial y^R}{\partial p} \right|_k \right) \frac{\left. \theta}{p} \frac{\partial p}{\partial \theta} \right|_k,$$

we get

$$\frac{\theta}{r} \frac{dr}{d\theta} \Big|_k = - \left\{ g \left(1 + \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k \right) + \frac{1}{\alpha_{2K}} \frac{k_2}{k_1 - k_2} \right\} \frac{\theta}{p} \frac{\partial p}{\partial \theta} \Big|_k < 0$$

if $k_1 \geq k_2$.

From (45) and (46), after substituting (24), we get

$$-g \frac{p}{y^R} \frac{\partial y^R}{\partial p} \Big|_k = \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k, \quad (56)$$

which, substituting in the previous equation, after some manipulations, follows

$$\frac{\theta}{r} \frac{dr}{d\theta} \Big|_k = \left(-g + \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} \right) \frac{\theta}{p} \frac{\partial p}{\partial \theta} \Big|_k. \quad (57)$$

Substituting (45) and (24) we get:

$$\frac{\theta}{k} \frac{dk}{d\theta} \geq 0 \text{ as } -g + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} - \frac{\alpha_{1L}}{(\alpha_{2K} - \alpha_{1K})^2} (\sigma_1 \alpha_{1K} + \sigma_2 \alpha_{2K} t^R) \geq 0. \quad (58)$$

D Welfare

D.1 Computation of $C_\theta(\rho)$

Differentiating (19) yields

$$\begin{cases} \frac{dk}{d\theta} = \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) \frac{dk}{d\theta} - \frac{dc}{d\theta} + \frac{dy_1}{d\theta} \Big|_{k,*} \\ \frac{dc}{d\theta} = c^* \gamma(c^*) \left(\frac{dr}{dk} \Big|_{\theta,*} \frac{dk}{d\theta} + \frac{dr}{d\theta} \Big|_{k,*} \right). \end{cases}$$

Given that

$$\int_0^\infty e^{-\vartheta t} \frac{dx(t)}{d\theta} dt = \int_0^\infty e^{-\vartheta t} \frac{d}{dt} \frac{dx(t)}{d\theta} dt = -\Delta x(0) + \vartheta X_\theta(\vartheta),$$

where $\Delta x(0)$ is the jump in the variable x right after the change in⁴⁰ θ , we can write the system

⁴⁰That is, $\Delta x(0) = \lim_{t \rightarrow 0^+} \frac{dx(t)}{d\theta}$.

above as

$$\begin{cases} \vartheta K_\theta(\vartheta) = \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) K_\theta(\vartheta) - C_\theta(\vartheta) + \frac{1}{\vartheta} \frac{dy_1}{d\theta} \Big|_{k,*} \\ \vartheta C_\theta(\vartheta) = c^* \gamma(c^*) \left(\frac{dr}{dk} \Big|_{\theta,*} K_\theta(\vartheta) + \frac{1}{\vartheta} \frac{dr}{d\theta} \Big|_{k,*} \right) + \Delta c(0), \end{cases} \quad (59)$$

where in the first line $\Delta k(0) = 0$ was used (capital is the state variable). Rearranging,

$$\begin{bmatrix} \vartheta - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) & 1 \\ -c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*} & \vartheta \end{bmatrix} \begin{bmatrix} K_\theta(\vartheta) \\ C_\theta(\vartheta) \end{bmatrix} = \begin{bmatrix} \frac{1}{\vartheta} \frac{dy_1}{d\theta} \Big|_{k,*} \\ c^* \gamma(c^*) \frac{1}{\vartheta} \frac{dr}{d\theta} \Big|_{k,*} + \Delta c(0) \end{bmatrix},$$

whose solutions are

$$C_\theta(\vartheta) = \frac{c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*} \frac{1}{\vartheta} \frac{dy_1}{d\theta} \Big|_{k,*} + \left[\vartheta - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) \right] \left[\frac{c^* \gamma(c^*)}{\vartheta} \frac{dr}{d\theta} \Big|_{k,*} + \Delta c(0) \right]}{\vartheta \left[\vartheta - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) \right] + c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*}}$$

$$K_\theta(\vartheta) = \frac{\frac{dy_1}{d\theta} \Big|_{k,*} - \Delta c(0) - \frac{c^* \gamma(c^*)}{\vartheta} \frac{dr}{d\theta} \Big|_{k,*}}{\vartheta \left[\vartheta - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) \right] + c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*}}.$$

The initial jump in consumption $\Delta c(0)$ still has to be computed. From the second equation in (59)

$$\Delta c(0) = -c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*} K_\theta(\mu) + \mu C_\theta(\mu) - c^* \gamma(c^*) \frac{1}{\mu} \frac{dr}{d\theta} \Big|_{k,*},$$

where μ is the positive eigenvalue of the linearized matrix. It must satisfy the characteristic equation:

$$\mu \left[\mu - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) \right] = -c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*}. \quad (60)$$

Hence, from the first equation in (59)

$$\begin{aligned} \frac{dy_1}{d\theta} \Big|_{k,*} &= \mu \left[\mu - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - \delta \right) \right] K_\theta(\mu) + \mu C_\theta(\mu) \\ &= -c^* \gamma(c^*) \frac{dr}{dk} \Big|_{\theta,*} K_\theta(\mu) + \mu C_\theta(\mu) \end{aligned}$$

so that

$$\Delta c(0) = \left. \frac{dy_1}{d\theta} \right|_{k,*} - c^* \gamma(c^*) \left. \frac{1}{\mu} \frac{dr}{d\theta} \right|_{k,*}.$$

Substituting for $\Delta c(0)$ into the expression for $C_\theta(\vartheta)$, and rearranging terms yields

$$\rho C_\theta(\rho) = \underbrace{\left. \frac{dy_1}{d\theta} \right|_{k,*}}_{\text{Tullock Effect}} + \underbrace{\frac{\mu - \rho}{\mu} \frac{\frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}}{\rho + \delta - \left. \frac{dy_1}{dk} \right|_{\theta,*} + \frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}}}_{\text{Habegger Effect}} \left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - \rho - \delta \right) \left. \frac{dk}{d\theta} \right|_*,$$

where $\left. \frac{dk}{d\theta} \right|_* = -\frac{\left. \frac{dr}{d\theta} \right|_{k,*}}{\left. \frac{dr}{dk} \right|_{\theta,*}}$ was used.

D.2 The Attenuation Factor

In this Appendix we show that the Attenuation Factor satisfies $0 \leq \text{AF} \leq 1$. Recalling that

$$\begin{aligned} \text{AF} &= \frac{\mu - \rho}{\mu} \frac{\frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}}{\rho + \delta - \left. \frac{dy_1}{dk} \right|_{\theta,*} + \frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}} \\ &\quad \frac{\frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*}}{\mu} < 0 \\ \mu &= \frac{1}{2} \left\{ \left. \frac{dy_1}{dk} \right|_{\theta,*} - \delta + \sqrt{\left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - \delta \right)^2 - c^* \gamma(c^*) \left. \frac{dr}{dk} \right|_{\theta,*}} \right\}, \end{aligned}$$

it follows that $\text{AF} > 0$ because $\mu - \rho \geq 0$ iff

$$\rho + \delta - \left. \frac{dy_1}{dk} \right|_{\theta,*} + \frac{c^* \gamma(c^*)}{\rho} \left. \frac{dr}{dk} \right|_{\theta,*} \leq 0.$$

If $\left. \frac{dy_1}{dk} \right|_{\theta,*} - (\rho + \delta) \geq 0$ it is a direct consequence of the definition of AF that $\text{AF} < 1$. If $\left. \frac{dy_1}{dk} \right|_{\theta,*} - (\rho + \delta) < 0$, using the characteristic equation (60), we can write AF as

$$\text{AF} = \frac{\mu - \rho}{\mu} \frac{\frac{\mu}{\rho} \left[\mu - \rho - \left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - (\rho + \delta) \right) \right]}{\frac{\mu}{\rho} \left[\mu - \rho - \left(\left. \frac{dy_1}{dk} \right|_{\theta,*} - (\rho + \delta) \right) \right] + \left. \frac{dy_1}{dk} \right|_{\theta,*} - (\rho + \delta)}$$

$$\begin{aligned}
&= \frac{\mu - \rho \frac{\mu}{\rho} \left[\mu - \rho - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) \right) \right]}{\mu \frac{\mu}{\rho} (\mu - \rho) - \frac{\mu - \rho}{\rho} \left(\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) \right)} \\
&= \frac{\mu - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) \right) - \rho}{\mu - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) \right)} < 1.
\end{aligned}$$

(Recall that $\mu - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) \right) - \rho > 0$ because $AF > 0$ and $\mu - \left(\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) \right) > 0$ when $\frac{dy_1}{dk} \Big|_{\theta,*} - (\rho + \delta) < 0$.)

D.3 Outside the Steady State

The impact of θ on welfare at any point in time is given by

$$\frac{dW}{d\theta} = \int_0^{\infty} e^{-\rho t} u'(c(t)) \frac{dc(t)}{d\theta} dt. \tag{61}$$

From the first equation in (19),

$$\frac{dk(t)}{d\theta} = \frac{\partial y_1}{\partial p} \Big|_k \frac{\partial p(t)}{\partial \theta} \Big|_k + \frac{dy_1}{dk} \Big|_{\theta} \frac{dk(t)}{d\theta} - \frac{dc(t)}{d\theta} - \delta \frac{dk(t)}{d\theta},$$

and hence

$$0 = \int_0^{\infty} e^{-\rho t} u'(c(t)) \left\{ \frac{\partial y_1}{\partial p} \Big|_k \frac{\partial p(t)}{\partial \theta} \Big|_k + \frac{dy_1}{dk} \Big|_{\theta} \frac{dk(t)}{d\theta} - \frac{dc(t)}{d\theta} - \delta \frac{dk(t)}{d\theta} - \frac{d}{dt} \frac{dk(t)}{d\theta} \right\} dt. \tag{62}$$

But

$$\int_0^{\infty} e^{-\rho t} u'(c(t)) \frac{d}{dt} \frac{dk(t)}{d\theta} dt = \int_0^{\infty} e^{-\rho t} u'(c(t)) \left(\rho + \frac{\dot{c}(t)}{\gamma(c(t))c(t)} \right) \frac{dk(t)}{d\theta} dt, \tag{63}$$

so substituting (63) and (62) in (61) gives:

$$\frac{dW}{d\theta} = \int_0^{\infty} e^{-\rho t} u'(c(t)) \frac{dc(t)}{d\theta} \left\{ \frac{\partial y_1}{\partial p} \Big|_k \frac{\partial p(t)}{\partial \theta} \Big|_k - \frac{dc(t)}{d\theta} + \left[\frac{dy_1}{dk} \Big|_{\theta} - \delta - \rho - \frac{c(t)}{\gamma(c(t))c(t)} \right] \frac{dk(t)}{d\theta} \right\} dt.$$

Using the Euler equation (18) we can write the last result as

$$\frac{dW}{d\theta} = \int_0^{\infty} e^{-\rho t} u'(c(t)) \left\{ \underbrace{\frac{dy_1(t)}{d\theta} \Big|_k}_{\text{Tullock Effect}} + \underbrace{\left(\frac{dy_1}{dk} \Big|_{\theta} - r \right) \frac{dk(t)}{d\theta}}_{\text{Habegger Effect}} \right\} dt,$$

where

$$\frac{dy_1(t)}{d\theta} \Big|_k = \frac{\partial y_1}{\partial p} \Big|_k \frac{\partial p(t)}{\partial \theta} \Big|_k.$$

Now, given that $\frac{dy_1(t)}{d\theta} \Big|_k = \frac{dy_1(0)}{d\theta} \Big|_k$, and that $\lim_{t \rightarrow \infty} \frac{dk(t)}{d\theta} < \frac{dk(t)}{d\theta} < 0$ if $k_1 \geq k_2$ it follows that

$$\frac{dy_1(t)}{d\theta} \Big|_k + \left(\frac{dy_1}{dk} \Big|_{\theta} - r \right) \frac{dk(t)}{d\theta} < 0 \text{ for any } t,$$

and, consequently,

$$\frac{dW}{d\theta} < 0 \text{ if } k_1 \geq k_2.$$

D.4 Signing $\frac{dy_1}{d\theta} \Big|_k + \left(\frac{dy_1}{dk} \Big|_{\theta} - r \right) \frac{dk}{d\theta}$ when $k_1 < k_2$

From (24) we have that:

$$\frac{dy_1}{d\theta} \Big|_k = \frac{\partial y_1}{\partial p} \Big|_k \frac{\partial p}{\partial \theta} \Big|_k = \frac{\partial y_1}{\partial p} \Big|_k \frac{p \alpha_g}{\theta \Delta}.$$

It follows from (55) that

$$- \left(\frac{dy_1}{dk} \Big|_{\theta} - r \right) \frac{dk}{d\theta} = \frac{y_1}{k} \frac{g}{\Delta} \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p \frac{dk}{d\theta}.$$

From (52) and (57) it follows that

$$\begin{aligned}\frac{dk}{d\theta} &= -\frac{-g + \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} \frac{\partial p}{\partial \theta} \Big|_k}{g \frac{\alpha_g}{1 - \alpha_g} + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} \frac{\partial p}{\partial k} \Big|_\theta} \\ &= \frac{k}{\theta} \frac{-g + \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}}}{g \frac{\alpha_g}{1 - \alpha_g} + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} (1 - \alpha_g) \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p},\end{aligned}$$

where we employed (23) and (24).

Consequently, we can write

$$\frac{dy_1}{d\theta} \Big|_k \geq - \left(\frac{dy_1}{dk} \Big|_\theta - r \right) \frac{dk}{d\theta}$$

whether

$$\frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k \geq g \frac{-g + \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}}}{g \frac{\alpha_g}{1 - \alpha_g} + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}}} \frac{1}{(1 - \alpha_g)},$$

or

$$(1 - \alpha_g) \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k \left(-g + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} \right) \geq g \left(-g + \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} \right).$$

Given that

$$\frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} = \frac{1 - \alpha_{1K}}{\alpha_{2K} - \alpha_{1K}} > 1 > g$$

the result follows.

E Comparing Competitive Rent-Seeking and Monopoly

E.1 Sector 1 Capital Intensive

We show that (see Figure 1)

$$\frac{k}{y_1} \frac{dy_1}{dk} \Big|_{\psi_2=0} = \frac{k}{y_1} \frac{\partial y_1}{\partial k} \Big|_p + \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k \frac{k}{p} \frac{dp}{dk} \Big|_{\psi_2=0} > 0, \quad (64)$$

where

$$\frac{k}{p} \frac{dp}{dk} \Big|_{\psi_2=0} = - \frac{\frac{k}{r} \frac{\partial r}{\partial k} \Big|_p}{\frac{p}{r} \frac{\partial r}{\partial p} \Big|_k} = - \frac{\alpha_{gg} (\theta y^R) \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p}{\alpha_{gg} (\theta y^R) \frac{k}{y^R} \frac{\partial y^R}{\partial p} \Big|_k - \frac{k_1}{f_1'} f_1'' \frac{\omega}{k_1} k_1' \frac{p}{\omega} \frac{d\omega}{dp}},$$

and the last equality comes from the expression for the interest rate,

$$r = (1 - g(\theta y^R(p, k))) f_1'(k_1(\omega(p))).$$

After substituting from (36), (37), (40), (41), and (42) it follows that:

$$\frac{k}{p} \frac{dp}{dk} \Big|_{\psi_2=0} = - \frac{\alpha_{gg} (\theta y^R) \frac{k}{y^R} \frac{\partial y^R}{\partial k} \Big|_p}{\alpha_{gg} (\theta y^R) \frac{k}{y^R} \frac{\partial y^R}{\partial p} \Big|_k + \frac{\alpha_{1L}}{\alpha_{1K} - \alpha_{2K}}}. \quad (65)$$

Hence (64) can be simplified (using (49)) to

$$\frac{k}{y_1} \frac{dy_1}{dk} \Big|_{\psi_2=0} = \frac{1}{\Gamma} \frac{k}{y_1} \left[-\alpha_{gg} (\theta y^R) \frac{1}{y_2} \frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k \left(\frac{\partial y_1}{\partial k} \Big|_p + p \frac{\partial y_2}{\partial k} \Big|_p \right) + \frac{\alpha_{1L}}{\alpha_{1K} - \alpha_{2K}} \frac{\partial y_1}{\partial k} \Big|_p \right],$$

where

$$\Gamma \equiv \alpha_{gg} (\theta y^R) \frac{k}{y^R} \frac{\partial y^R}{\partial p} \Big|_k + \frac{\alpha_{1L}}{\alpha_{1K} - \alpha_{2K}} > 0 \text{ if } k_1 > k_2.$$

The inequality in (64) follows from (50).

E.2 Sector 1 Labor Intensive

Note that if $k_1 < k_2$ then

$$\alpha_{gg} (\theta y^R)_k \frac{p}{y_2} \left(\frac{\partial y_1}{\partial k} \Big|_p + p \frac{\partial y_2}{\partial k} \Big|_p \right) > \frac{\alpha_{1L}}{\alpha_{2K} - \alpha_{1K}} \frac{\partial y_1}{\partial k} \Big|_p.$$

Consequently, using (50) and (49) in (65) we have

$$\frac{k}{p} \frac{dp}{dk} \Big|_{\psi_2=0} \leq - \frac{\frac{k}{y_1} \frac{\partial y_1}{\partial k} \Big|_p}{\frac{p}{y_1} \frac{\partial y_1}{\partial p} \Big|_k} = \frac{k}{p} \frac{dp}{dk} \Big|_{y_1} \text{ as } \Gamma \geq 0, \text{ or as } \frac{k}{p} \frac{dp}{dk} \Big|_{\psi_2=0} \leq 0.$$

The fact that

$$\left. \frac{k}{p} \frac{dp}{dk} \right|_{\psi_2=0} \neq 0$$

guarantees that $y_1 = \text{constant}$ and $\psi_2 = 0$ intersect only once (see Figure 2.)

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