R&D Investment and International Location in Oligopolist Industries*

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Abstract

We analyze the influence of R&D investment on firms and skilled workers location decisions. Contrary to most 'new' economic geography literature, based on the Dixit-Stiglitz monopolistic competition model, we use instead a Cournot oligopoly model where firms perform R&D investment to achieve future reductions in marginal costs. We find 'home market' and 'competitiveness effects' resulting from R&D investment: firms located in countries with more skilled-labor perform higher levels of R&D, and as such they are more competitive than competitors in other countries. This 'R&D investment linkage' effect can lead to agglomeration and cumulative causation effects. Firms want to locate where skilled-workers are to become more competitive; while skilled-workers want to locate where firms are, to attain higher welfare levels, and to work in the country that gives them more chance to use their skill.

Keywords: Economic Geography, Oligopoly, R&D Investment, Agglomeration, R&D Linkage Effects, Brain Drain, Delocalization.

JEL Classification: F12, L13, R12.

1 Introduction

The starting point of this paper, is the so-called 'New Economic Geography' (NEG) developed during the 90's by people as Krugman (1991), Venables (1996), and Fujita et al. (1999), just to mention a few. The features of NEG can be

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summarized as follow: monopolistic competition, 'iceberg' trade costs, 'home market' and 'price index' effects, and agglomeration effects. Monopolistic competition type of modeling follows directly from the very influential paper by Dixit and Stiglitz (1977), while 'iceberg' trade costs were first introduced by Samuelson (1954). The 'home market' and 'price index effects' are trade-marks of the 'new' trade theory models (see for example Krugman, 1980)[1]. The new feature of NEG (besides the combination of the aspects mention above) is the existence of agglomeration effects (i.e.: the propensity for economic activity to concentrate in few locations) that arise endogenously for certain parameter values.

Undoubtedly, NEG contributed greatly to bring back the questions of space to the core of economic profession. But besides the virtues, some limitations can also be point out, being the most important associated with the use of special functional forms, namely monopolistic competition. Neary (2001) makes a complete assessment of these limitations that can be summarized as follow. In first place, Dixit-Stiglitz monopolistic competition model ignores any type of interdependence between firms since firms almost neglect the existence of their competitors when making their strategic choices (i.e.: prices are always a constant mark-up over marginal costs). As consequence, firms do not act strategically in the market-place, since firms strategic choices (prices in monopolistic competition) are not used to influence competitors choices. Firms also do not influence market behavior since they do not commit inter-temporally (i.e.: dynamically this means that firms behave always the same way in every period since they do not take actions that can affect future performances as is the case with R&D investment). Furthermore, there is a equivalence between labor market spatial equilibrium and firm market spatial equilibrium (i.e.: firm migration decisions mimic exactly workers migration patterns since they are connected through the labor market clearing condition). Ideally, we should allow them to not be so directly related, to see the interconnections between the two, and also because what motivates workers to migrate may be not always the same what induces firms to delocalize.

Our paper tries to take in consideration some of the limitations pointed out above. Namely, we abandon Dixit-Stiglitz monopolistic competition model to use instead a Cournot oligopoly model in the line of Brander (1981),where firms perform R&D investment to reduce marginal costs as in Spencer and Brander (1983). Our formalization strategy has some advantages in relation to the Dixit-Stiglitz one. In first place, it allows for the existence of economic interactions between firms. In fact when making their strategic choices, firms try to influence competitors behavior. This is mirrored for example when firms choose output levels, that depend on the level of competition in the market place.

Second, firms act strategically in two strategic variables: R&D investment and output. Firms invest in R&D in the pre-market stage, while in the market-

[1] The 'home market effect', states that countries with higher demand have a proportionally larger share of industry. The 'price index effect', states that countries with more industry have a lower cost of living (i.e.: a lower price index) since, while imports incur in transport costs (increasing prices to final consumers), home-produced varieties do not.

stage firms compete in outputs. As in Spencer and Brander (1983) and Leahy and Neary (1996), R&D investment has as objective future reductions in marginal costs. As consequence of this, firms influence market behavior since they commit inter-temporally, in the sense that the choices in one period will affect future choices and performances by firms.

In third place, location decisions by firms and workers are taken separately: migration by workers arise in response to indirect utility differentials between locations, while firms choose locations where they can get higher profits. Conversely, profit opportunities in a location can result from both the demand side (bigger home market triggered by labor migration patterns, as in NEG), but also from the supply side: firms are more efficient in the location where a higher level of R&D investment is performed, and in our model this location happens to be the one where more skilled-workers are located. Being that so, then firms in this country will be more competitive than firms in other countries (since due to R&D investment they achieve lower levels of marginal costs). This can lead to delocalization of firms to the country that host a higher share of skilled-labor. We call this the 'R&D linkage effect'. But in another turn, skilled-workers prefer locations where more firms are located, because prices are lower here (and therefore utility is higher) and they can more easily be employed in the sector that use their skill. In the end interconnections between delocalization decisions by firms and migration decisions by skilled-workers can promote agglomeration of economic activity in result of cumulative causation effects.

In this sense, we study the influence of R&D investment on location decisions of firms and workers. By doing this, first we also test NEG against a different market structure, namely oligopoly; and second we look at other firms strategies than only prices and outputs, namely R&D investment. Other issues analyzed that will deserve more attention are the problematic of 'brain drain' of skilled-workers to more advanced countries, and the issue of delocalization that can give origin to phenomena of regional industrial growth and decline.

Besides this section, this paper has more ten sections. In the next section, we introduce the base-line model. The model has two cases: the standard Cournot model (from now on SCM) and the R&D investment model (from now on RDM). In the SCM firms only compete in quantities, while in the RDM firms compete in both outputs and R&D investment levels. Then, we present the production equilibrium of both models. In the fourth and fifth section, we explore the agglomeration and dispersion forces present in the model. After, we look at delocalization phenomena by firms, and skilled-workers migration decisions. Then, we study the stability of the agglomerated equilibrium, and the spatial equilibriums associated with the structural relations and parameters in our model. We conclude by discussing the results.

2 The Model

The model considers two regions/countries[2], two sectors and two factors of production. The regions are home (H) and foreign (F), where foreign variables are denoted by an asterisk. The two sectors are the oligopolist/increasing returns hi-tech sector (HTS) and the perfect competition/constant returns traditional sector (TS)[3]. The goods are the oligopolist sector good (hi-tech good) and the constant returns sector good (traditional good). Furthermore, the TS-good can be freely trade between regions without having to incur in any type of costs (namely transport costs), while the HTS-good is subject to ad-valorem trade costs (t) when exchange between different regions[4].

The two factors of production are unskilled-labor (A) and skilled-labor (L). Both factors are sector-specific: factor A can only be employed in the TS, while factor L in the HTS. Furthermore, the factor A is immobile between regions, and is distributed evenly between regions (i.e.: both home and foreign have $\frac{A}{2}$ units of unskilled-labor). On the contrary, the factor L is perfectly mobile between regions and u ($u \in (0, 1)$) denotes the share of this factor located at H (and $(1 - u)$ at F). As such there is uL skilled-workers at home and $(1 - u) L$ skilled-workers at foreign. Below, we will explain how these factors are used in production.

Make also $M = A + L$ the total number of workers/consumers (skilled plus unskilled-workers)[5] in the world economy. Denominating r as the share of workers at H (and consequently $(1 - r)$ the share of consumers at F); then $rM = \frac{A}{2} + uL$ is the number of workers at H; while $(1 - r)M = \frac{A}{2} + (1 - u) L$ is the number of workers at F. This means, that r is never equal to zero, since there are always a percentage of world workers that are immobile (i.e.: unskilled-labor). Furthermore:

$$\begin{aligned} r &= \frac{\frac{A}{2} + uL}{A + L} \\ (1 - r) &= \frac{\frac{A}{2} + (1 - u) L}{A + L} \end{aligned} \tag{1}$$

Then r only changes with u (since only skilled-labor moves between locations) and rM is linear in uL. Due to this fact through the paper, and for

[2] Through the paper we use indifferently regions and countries. We will argue below that, contrary to Krugman (1991) and Krugman and Venables (1995), our model can be applied to both regional and international contexts. Krugman model is more adequate for regional contexts because workers have no restrictions to migration. On the contrary, Krugman and Venables model is more adapted for international contexts since workers can not move between locations.

[3] Through the paper, the traditional sector is kept on the background. The role of this sector is to represent what can be interpreted as 'the rest of the economy', and also (as it will be seen bellow) to assure that whatever agglomeration phenomena in place, a region always keeps some economic activity.

[4] Note that this differs from NEG monopolistic competition models where trade costs are of the 'iceberg' type.

[5] In our model workers (skilled and non-skilled) are both labor force and consumers.

simplification purposes, we sometimes work with rM instead of $\frac{A}{2} + uL$, but when deemed necessary we show results in terms of u, L and A.

Consider N to be the total number of firms in the HTS in the world economy, and $sN = n$ the number of firms at H (therefore $(1-s)N = n^*$ is the number of firms at F). As such, s is the share of firms at H, and $(1-s)$ is the share of firms at F.

We consider quasi-linear Preferences in the two goods with a quadratic sub-utility in the good produced by the oligopolist sector:

$$U = aQ - \frac{b}{2}Q^2 + q_0 \tag{2}$$

And similarly for the foreign country. Where, $Q = \sum_{i=1}^{n} q_i + \sum_{i=1}^{n^*} x_i^*$ is the total home consumption of the hi-tech sector good. The variables of interest are q (sales of a representative H firm in the H market), x (exports by a representative H firm to the F market), q^* (sales of a representative F firm in the F market), x^* (exports by a representative F firm to the H market), and q_0 (production and consumption of the traditional good).

Each individual is endowed with a unit of labor (A or L), and $\bar{q}_0 > 0$ units of the traditional good[6]. Then consumers have the following budget constraint:

$$PQ + q_0 = I + \bar{q}_0 \tag{3}$$

Where, P are prices at home, and I is income. From this maximization problem we can get the indirect demand. To do this, first solve for q_0 in (3) and substitute in (2), then calculate the first order condition to obtain:

$$P = a - bQ \tag{4}$$

The direct demand is obtained by solving for total sales:

$$Q = \frac{a}{b} - \frac{1}{b}P \tag{5}$$

To find the indirect utility function just substitute equation (5) in equation (2), and substitute also for q_0 from (3):

$$V = \frac{a^2}{2b} - \frac{a}{b}P + \frac{1}{2b}P^2 + I + \bar{q}_0 \tag{6}$$

Turning now to firms, we define total profits by a representative H firm as:

[6] This assumption is made in order to assure that the consumption of the traditional good is always positive (see Ottaviano et al., 2001).

$$\Pi_i = (P - C_i)\, q_i\,(rM) + (P^* - C_i - t)\, x_i\,((1-r)\,M) - \Gamma_i \tag{7}$$

Where t are the trade/transport costs[7], C the marginal costs, and Γ the fixed costs. The structure of both C and Γ will be explained below. In our model delocalization by firms is triggered by profit opportunities in different locations, i.e.: firms delocalize to regions where profit opportunities are higher. There are therefore spatial phenomena of the type regional 'growth' and 'decline'. Then if agglomeration arises, delocalization is the result of industry dynamics in different countries.

We consider two cases related with the nature of the production game played by firms. The first case, is the what we call standard Cournot model (see Brander, 1981). In this game firms play a simultaneous one shot game in quantities. The second case is the Cournot model with R&D Investment (see Spencer and Brander, 1983 for the duopoly case; and Leahy and Neary, 1996 for the duopoly case with n-sectors)[8]. In the RDM firms play a two stage game in the production phase, where in the first stage firms choose R&D levels and in the second stage firms compete in quantities (*a la* Cournot).

We turn now to technology in the TS and HTS. In the TS, technology requires 1 unit of A to produce one unit of output. Then, since the good produced in the TS is freely trade, it implies that in equilibrium unskilled-labor wages are equal to one in both regions (i.e.: $w_A = w_A^* = 1$).

To define technology in HTS we need to explain the structure of C (marginal costs) and Γ (fixed costs). In both cases analyzed (SCM and RDM), technology in the HTS implies that marginal costs and fixed costs are incurred only in terms of skilled-labor. In our central case (i.e.: the R&D model), we define technology in the HTS as:

$$C_i = (c_i - \theta k_i)\, w_L \tag{8}$$

$$\Gamma_i = \left(\gamma \frac{k_i^2}{2} + f_i\right) w_L \tag{9}$$

Where, c_i are the marginal costs without R&D investment (i.e.: initial marginal costs), f_i are the operational fixed costs, w_L is the wage rate in the increasing returns sector, k is the level of R&D investment conducted by H firms, θ is a parameter that indicates the cost-reducing effect of R&D investment, and γ measures the cost of R&D investment. This type of R&D investment is also

[7] We use the terms trade and transport costs interchangeably through the paper.

[8] Note that, skilled-workers in the SCM do not have any special type of skill when compared to the skilled-workers in the RDM (i.e.: they not perform R&D investment). Then, is not accurate to think of them as skilled labor, instead, and as in Krugman (1991), is more appropriate to designate them as industrial workers. In the same fashion, the good produced by the oligopolist sector in the SCM can hardly be thought as a hi-tech good. The same applies to consider the oligopolist sector in the SCM as a hi-tech sector. Therefore, it is more correct to think of the oligopolist sector in the SCM as a industrial sector that produces a industrial good using as only input industrial workers, under increasing returns to scale.

known as cost-reducing R&D investment, since R&D reduces marginal costs. Then, R&D investment has two main characteristics: first, reduces marginal costs; second, increases fixed costs. The effects of R&D on the competitiveness of a firm depends therefore on the balance between marginal and fixed costs: the first increases competitiveness the second reduces profitability.

For simplification purposes we assume that the skilled-labor wages are fixed in both regions and equal to unskilled-labor wages. As such we normalize both skilled and unskilled-labor wages to unity, i.e.: $w_A = w_A^* = w_L = w_L^* = 1$[9] (this gives a partial equilibrium structure to our model). This simplification is done for one additional reason. We said in the beginning that we also want to analyze the problematic of 'brain drain'. One obvious reason for 'brain drain' to exist is wage differentials: a software programer emigrates from India to Germany, because he is better paid in Germany. By assuming that wages are fixed and not differ from country to country, we want to isolate the income factor, in order to see if even when wage differentials are non-existent the problematic of 'brain drain' still arises[10].

We consider that the SCM is a special case of the RDM, when both θ and γ are zero. Therefore in the SCM we have:

$$\begin{aligned} C_i &= c_i w_L \\ \Gamma_i &= f_i w_L \end{aligned} \tag{10}$$

For simplification purposes we also assume that the skilled-labor wages are

[9] This assumption can be justified if we think of two countries in the same stage of development (for example Holland versus Belgium), since at this level wage differentials are smaller. At the regional level this can also happen since most OECD countries follow the rule: "same job, same payment". As corollary our model can hardly be applied in between countries with big wage differentials (e.g.: North-South countries) and/or countries with big regional wage differentials. One other question can arise: is our model adequate for the US-EU reality? Bertola (1999) summarizes the differences between US and EU labor markets: EU labor markets are characterized by limited labor migration and compressed regional wage differentials when compared with US. At the aggregate level, however, wage levels in US and EU do not differ significantly. Therefore, we think that the model can be applied in between European countries, regions within European countries, and in between US and EU, but it is of more difficult application within US regions since at this levels wage differentials play a crucial role in migration dynamics (see Blanchard and Katz, 1992). However, in our opinion, the question is not only if there are (or not) wage differentials, but if these wage differentials are sufficiently big to trigger migration by workers. Take the example of Western Europe; in spite of agreeing that impediments to migration are bigger in Europe than in US, we think that in Europe workers also do not migrate so much as in US because wage differentials in Europe are smaller. As a counter-factual for this argument, remember the migration movements from southern European countries (as Italy, Portugal and Spain) to northern European countries (as France and Germany) during the 60's and 70's. At this time impediments to migration were even higher than are now, but wage differentials were sufficiently high to promote migration.

[10] As we have said before, only skilled-labor can migrate. In the base of our argument is that skilled-labor is more mobile than unskilled-labor. Therefore, we do not want to stress that unskilled-labor does not migrate but that is less mobile than skilled-labor. Furthermore, since it is also acknowledged that skilled-labor is also more mobile at the international level, this defends our argument that our model can also be applied to international contexts. Note that in the RDM this is not only a question of language, skilled-workers are not only skilled-workers because we said so, but because they perform R&D investment.

fixed in both regions and are equal to the unskilled-labor wages. This means once again that $w_A = w_A^* = w_L = w_L^* = 1$. As such also the SCM model is in the partial equilibrium version.

Furthermore, since all firms are equal (symmetry assumption) then in equilibrium in both SCM and RDM, for all i and j (from home and foreign), $c_i = c_j = c$, and $f_i = f_j = f$. Then in the SCM we have $C_i = C_j = C$, and $\Gamma_i = \Gamma_j = \Gamma$. Note however that in the RDM, C_i does not necessarily equals C_j and the same for Γ_i and Γ_j[11]. This happens to be so, because if firms perform different levels of R&D investment they can also endogenously differentiate themselves at the level of marginal and fixed costs (since both depend on R&D levels).

Now we turn to the timing of the game. In stage 1 skilled-workers and firms make their location decisions. In stage 2 firms choose R&D levels (pre-market stage), and finally in stage 3 firms make the production decisions (and workers/consumers their consumption decisions)[12]. We call stage 1 the location game, while stage 2 and 3 the production game.

The game as usual is solved by backward-induction. Regarding this, it can be useful to think of our game also as a temporal game with $T = 1, 2, ...\Upsilon$ periods (i.e.: a finitely repeated game). Bellow, we will explain in more detail how the temporal dimension is introduced. As such, we solve the model for any distribution of firms and skilled-workers[13], i.e.: $s \in (0, 1)$ and $u \in (0, 1)$[14]. This means, that we present expressions for outputs and R&D investment associated for any distribution of firms and workers. Given these expressions, we can also study what location patterns of firms and skilled-workers are equilibriums.

Just one more final remark about the model presented above. As we have said in the introduction, the type of modelling adopted allows us to analyze two important issues: the question of 'brain drain' of skilled-workers; and the dynamics involved in the phenomena of delocalization of firms. In the first case, by assuming that only skilled-labor can move between locations we try to isolate some of the reasons that leads this type of labor to migrate even when wage differentials between locations are inexistent or negligible. In the

[11] In practice, and since in these type of models firms are symmetric at the national level (that is why these models are also called 'national market games'), C_i can only be different of C_j^* (i.e.: $C_i \neq C_j^*$, but $C_i = C_j$), and the same for Γ_i and Γ_j^* (i.e.: $\Gamma_i \neq \Gamma_j^*$, but also $\Gamma_i = \Gamma_j$). Note however, that in the SCM firms are only symmetric at the national level because of transport costs (at the level of marginal costs and fixed costs, home and foreign firms are, unless otherwise assumed, symmetric). In the RDM, national and foreign firms can be unsymmetrical, not only because of transport costs but also because of R&D investment. As it will be seen bellow, in this model asymmetries between firms (at the level of marginal costs and fixed costs) arise as a result of asymmetric labor markets.

[12] As it must be obvious, in the SCM there are only two stages since firms do not perform R&D investment: in stage 1 industrial workers and firms choose location; in stage 2 firms choose output levels.

[13] Therefore, and contrary to what is usual in the NEG literature, in that is assumed that workers initially are evenly distributed (see Krugman, 1991), we do not consider any initial exogenous distribution of workers and/or firms.

[14] Note that due to this, we do not need to index the equations by time, since they are valid for any period of the game, once they are expressed for any distribution of skilled-workers and firms.

second case, we look at delocalization as a dynamic spatial phenomena since it depends on the comparative performance of firms in different locations and is influenced by both the location of skilled-workers and R&D investment. In the following sections we will look at these two issues and show that they are closely inter-linked.

3 Production Game

We solve the game by backward induction, starting therefore by the production game. In the next sections we solve the location game. Remember that in the SCM the production game is just to find the expressions for outputs (and prices); while in the RDM besides finding the solution for outputs (stage 3) we also solve for R&D investment levels (stage 2). Note also, that the production game depends on both the spatial distribution of firms and workers, as such we derive it for any $s \in [0, 1]$, and $u \in [0, 1]$. Only in the following sections, we analyze for what s and u the location game (and in consequence also the production game) is a stable equilibrium.

Besides the expressions of outputs and R&D investment we will also present the expression for prices and also, what is known as, the overlapping market condition (OMC), first introduced by Brander (1981). Prices are a central variable in our model since wages are fixed, and being that so, location choices of skilled-workers (determined by indirect utility differentials between countries) are only based on prices differences at home and foreign. On the other side, the OMC gives the threshold level of market size (or trade costs) that makes trade profitable for firms. Bellow this threshold level we are in autarchy: exports are too costly for firms; above this threshold trade is possible: exports are profitable.

As showed by Brander (1981), the OMC from the side of the foreign firm is obtained by setting $x^* = 0$, at $s = 1$ and solving for D. Therefore, this condition states the minimum market size necessary for a potential foreign firm be able to export profitably to the home country when home hosts all the firms. However, this condition can also be defined from the point of view of the home firm. The correspondent OMC from the side of the home firm is obtained by setting $x = 0$ at $s = 0$, and then also solve for market size. The interpretation is the same as before for the OMC from the side of the foreign firm. We will see below, that under the SCM this condition is the same for both home and foreign firms. However, under the RDM that is not the case anymore. This happens, because firms can endogenously become asymmetric as result of R&D investment, and being that so home and foreign firms have different levels of market access to international markets (i.e.: they have different OMC).

By looking at the OMC (of both the SCM and RDM) we want to see the relation between trade and location. In fact we will see in subsequent sections that when trade is possible agglomeration of economic activity is promoted. When trade is restricted or forbidden dispersion is preferred by both firms and skilled-workers[15].

3.1 Standard Cournot Model

In this section we derive expression for outputs, prices, and OMC for the SCM.

Outputs The output equilibrium in the SCM is:

$$\begin{aligned} q &= \frac{D+(1-s)\,Nt}{b\,(N+1)} \\ x &= \frac{D-((1-s)\,N+1)\,t}{b\,(N+1)} \\ q^{*} &= \frac{D+sNt}{b\,(N+1)} \\ x^{*} &= \frac{D-(sN+1)\,t}{b\,(N+1)} \end{aligned} \tag{11}$$

Where $D = a - c$ is a measure of market size.

Prices The prices at H and F associated with the level of outputs defined above, are:

$$\begin{aligned} P &= c + \frac{D+(1-s)\,Nt}{N+1} \\ P^{*} &= c + \frac{D+sNt}{N+1} \end{aligned} \tag{12}$$

Overlapping Markets Condition The OMC under the SCM from the perspective of a representative foreign firm is (setting $x^{*}=0$, at $s=1$ and solving for D)[16]:

$$D > t\,(N+1) \tag{13}$$

As such, in the SCM, the OMC gives the relation between market size (D), trade costs (t), and the total number of firms in the world economy (N), that makes trade profitable for an individual firm. Trade is restricted the higher the trade costs and the larger the number of firms in the world economy. The role of trade costs is self explanatory: higher trade cost makes exports more costly. More firms means more competition, and therefore less output per firm, and this in turn less profitability of exports. A higher market size allows firms to export even when trade costs and competition penalizes exports.

[15] For a more deep analysis of the relations between the OMC and international trade, see however, Pires (2003).

[16] It can be easily check, that the same condition applies for a representative home firm (just set $x=0$, at $s=0$ and solve for D).

3.2 R&D Investment Model

In this section, we derive expression for outputs, R&D investment levels and prices for the RDM. As before we also show the OMC.

Outputs and R&D Investment We solve the production game in the RDM by backward induction, i.e.: we find first the output stage equilibrium and only then the R&D investment stage equilibrium. The output stage equilibrium is:

$$\begin{aligned}
bq &= \frac{D + (1-s)\,Nt + \theta k\left((1-s)\,N + 1\right) - (1-s)\,N\theta k^{*}}{N+1} \\
bx &= \frac{D - \left((1-s)\,N + 1\right)t + \theta k\left((1-s)\,N + 1\right) - (1-s)\,N\theta k^{*}}{N+1} \\
bq^{*} &= \frac{D + sNt + \theta k^{*}\,(sN+1) - sN\theta k}{N+1} \\
bx^{*} &= \frac{D - (sN+1)\,t + \theta k^{*}\,(sN+1) - sN\theta k}{N+1}
\end{aligned} \tag{14}$$

The first thing to note is that outputs increase with the level of local R&D investment (k)[17] weight by the number of foreign firms plus one, and decreases with foreign R&D investment (k^*) weight by the number of foreign firms. Then, the assumption of symmetry between national firms, leads to 'national market games' type of outcomes, i.e.: home firms benefit from positive performance from other national firms (and conversely from negative performance from foreign firms)[18].

To solve for the R&D investment stage equilibrium, we suppose that there is no strategic R&D investment. In practice this means, that firms make their output and R&D investment decisions simultaneously. This hypothesis, made for simplification purposes, does not affect our primary goal that is to see the role of R&D investment in influencing firms location decisions. The only thing that we ignore is the possibility that firms over (or under) invest to influence the final market outcome. We are therefore, analyzing the relation between R&D investment and location patterns when firms invest at the optimal social market level[19]. Then the R&D investment stage equilibrium is simply:

[17] Since $k_i = k_j = k$ for all $i,\ j \in n$.

[18] Note that this happens even without assuming R&D spillovers or other type of external economies.

[19] The question of strategic investment only matters in these issues when firms have different commitment power, i.e.: some firms can make strategic investment while others can not. When this is the case, asymmetry between firms arises affecting the comparative relation between firms in the market place (i.e.: some firms are more 'powerful' than others). When firms have the same commitment power (i.e.: wether all firms can make strategic R&D investment, or all firms can not), only the level of R&D investment is affected (i.e.: when firms strategically invest, they either over- or under-invest, and when they do not strategically invest they invest at the optimum social level). However, the important point is that if all firms have the same commitment power then firms perform the same level of R&D investment since they are symmetric. Then also the comparative economic relations between existing firms is not altered, only the level of this relation is affected.

$$\begin{aligned}\gamma k &= \theta M\left(qr + x\left(1-r\right)\right) \\ \gamma k^{*} &= \theta M\left(q^{*}\left(1-r\right) + x^{*}r\right)\end{aligned} \tag{15}$$

It can be easily seen that R&D increases with total world population of consumers/workers (M) and the cost-reducing effect of investment (θ); and decreases with the cost of R&D investment (γ). Furthermore, and most importantly, the levels of R&D investment depend on the size of the local and foreign labor market: it increases with local share of skilled-labor and decreases with the foreign share of skilled-labor. Since in these type of models, firms depend more on local sales than exports, it is expected that in the short-run an increase in the number of local skilled-workers also increases the total level of R&D investment by firms located in that country (bellow we will established conditions for this to be so).

The model can be solved simultaneously for q, q^{*}, x, x^{*}, k, and k^{*}. For simplification we set $b = 1$. Even so, the RDM gives quite cumbersome expressions for output variables. The only exception is the expression for the level of R&D investment. Given this, the strategy that we are going to follow is to present explicitly only the expression for R&D investment (but not the one for outputs)[20].

In the case of the level of R&D investment by a representative home and foreign firm we have respectively:

$$\begin{aligned}k &= \frac{\theta M}{\phi}\left(\left(\theta^{2}M-\gamma\right)\left(D-t\left(1-r\right)\right)-2\gamma Nt\left(1-s\right)\left(r-\frac{1}{2}\right)\right) \\ k^{*} &= \frac{\theta M}{\phi}\left(\left(\theta^{2}M-\gamma\right)\left(D-rt\right)+2\gamma sNt\left(\left(r-\frac{1}{2}\right)\right)\right)\end{aligned} \tag{16}$$

With $\phi = \varphi\left(\theta^{2}M-\gamma\right)$, where $\varphi = \left(\gamma\left(N+1\right)-\theta^{2}M\right)^{21}$. As proved in the appendix for having $k > 0$, we need two conditions to be satisfied. The first one, is that the determinant of the system implied by the R&D investment equations is positive. This implies[22]:

$$\gamma > \theta^{2}M \tag{17}$$

We call this condition the 'determinant condition'. Then, this implies that $\phi < 0$, and $\varphi > 0$.

[20] The expressions for the other variables can be obtained upon request to the author.

[21] Note that, q and x (and q^{*}, x^{*}) also have ϕ as denominator.

[22] Note that, the condition $\gamma > \theta^{2}M$ implies that the cost of R&D investment and the cost-reducing effect of R&D are closely linked. The parameter γ is only high if it is so in relation to θ. The reverse also holds. For example if $M = 100$; $\gamma = 2550$ is high if $\theta = 1$ (since the threshold level $\gamma = \theta^{2}M$ is 100), but it is low if $\theta = 5$ (because in this case the threshold level is 2500). Therefore, what matters is not the absolute value of γ or θ, but their relative value.

In the case of home R&D investment levels, for having $k > 0$, we also need market size to be such that:

$$D > \frac{t}{\theta^2 M - \gamma}\left(\theta^2 M\left(1-r\right) + \gamma\left(2N\left(1-s\right)\left(r-\frac{1}{2}\right) - \left(1-r\right)\right)\right) \tag{18}$$

We call this the 'R&D condition'. In appendix we show that, as long as, $\gamma > \theta^2 M$, this condition is always satisfied for $r > \frac{1}{2}$ (and as such also for $u > \frac{1}{2}$), since the expression on the right side becomes negative, and D is positive. For $r < \frac{1}{2}$ (i.e.: for $u < \frac{1}{2}$), we need market size to be sufficiently big for this condition to be satisfied.

In the case of foreign R&D investment this condition is just symmetric[23]:

$$D^* > \frac{t}{\theta^2 M - \gamma}\left(\theta^2 M r - \gamma\left(2sN\left(r-\frac{1}{2}\right) + r\right)\right) \tag{19}$$

Then, also as long as, $\gamma > \theta^2 M$, this condition is always satisfied for $r < \frac{1}{2}$; while for $r > \frac{1}{2}$ we need market size to be sufficiently big for this condition to be satisfied. This means that the country with higher share of skilled-labor performs always positive levels of R&D investment independently of market size.

Prices We can also solve for equilibrium prices. The general expression is:

$$\begin{aligned} P &= c + \frac{D + (1-s)\,Nt - \theta N\left(sk + (1-s)\,k^*\right)}{N+1} \\ P^* &= c + \frac{D + sNt - \theta N\left(sk + (1-s)\,k^*\right)}{N+1} \end{aligned} \tag{20}$$

Note that, prices benefit from R&D investment of both H and F firms (i.e.: prices decrease with k and k^*). R&D investment in this sense is world welfare improving since it diminishes prices in both countries (but of course it diminishes more the prices in the country where more firms are established, since consumers do not have to pay the associated extra transport costs).

We can also give the explicit expressions for home and foreign prices:

$$\begin{aligned} P &= c + \frac{\gamma(N+1)(D+Nt(1-s)) - \theta^2 M\left(D(N+1) - 2Nt(1-r)\left(s-\frac{1}{2}\right)\right)}{\varphi(N+1)} \\ P^* &= c + \frac{\gamma(N+1)(D+sNt) - \theta^2 M\left(D(N+1) + 2rNt\left(s-\frac{1}{2}\right)\right)}{\varphi(N+1)} \end{aligned} \tag{21}$$

[23] We denote market size by an asterisk to stress that this is the R&D condition from the side of the foreign firm. Note that we do not intend to mean that the foreign market has a different market size. We continue to assume symmetry between countries at this level.

Overlapping Markets Condition The OMC under the RDM, from the side of a representative foreign firm, is (setting as before $x^* = 0$, at $s = 1$ and solving for D)[24]:

$$D^* = t\,(N+1) + N\theta k - \theta k^*\,(N+1) \tag{22}$$

In the RDM, the OMC gives the relation between market size, trade costs, and R&D investment by H and F firms that makes trade possible (i.e.: profitable). It can also be seen that the OMC (for the foreign firm) increases with trade costs, and the level of R&D investment by H firms, while it decreases with the level of R&D investment by foreign firms. This comes at no surprise, since the more the home firms invests more difficult foreign firms will find to export to the home market. Conversely, the more the foreign firms invest more easily they will find to export to the home market.

Contrary to what happens in the SCM, in the RDM the OMC from the perspective of the home firm is not equal to the OMC from the perspective of the foreign firm. In fact the OMC from the perspective of the home firm is:

$$D = \left(t\,(N+1) + N\theta k^* - \theta k\,(N+1)\right)$$

This increases as before with trade costs and the total number of firms in the world economy, but now the role of R&D investment by home and foreign firms is inverted. The OMC from the perspective of the home firm, increases with foreign R&D investment and decreases with home R&D investment. Then firms from different countries can have different levels of access to international markets, depending this on R&D investment made by local firms.

The explicit expressions for both OMC, from the perspective of the foreign firm and home firm respectively are:

$$D^* = t\frac{\theta^2 M\left(\theta^2 M(2N+1)(1-r)-\gamma\left(2\left(N^2+1\right)+5N-r(2N(N+2)+1)\right)\right)+\gamma^2(N+1)^2}{\gamma(N+1)(\gamma-\theta^2 M)} \tag{23}$$

$$D = t\frac{\gamma^2(N(N+2)+1)-\gamma\theta^2 M(r(2N(N+2)+1)+(N+1))+r\theta^4 M^2(2N+1)}{\gamma(N+1)(\gamma-\theta^2 M)} \tag{24}$$

4 Agglomeration Forces: Demand and R&D Investment Linkage Effects

Before proceeding to analyze the location game, in this section we take a preliminary step. We will look at what forces are in play that can promote agglomeration of economic activity. As it is known, NEG explores mainly two mechanisms of agglomeration, namely international labor migration (Krugman, 1991); and

[24] We denote market size by an asterisk to stress that this is the OMC from the side of the foreign firm. Note that we do not intend to mean that the foreign market has a different market size. We continue to assume symmetry between countries at this level.

input-output vertical relations between firms (Krugman and Venables, 1995). In both cases 'demand' and 'cost linkage effects' are at work. In the international labor migration case, the 'demand linkage effect' operates as the name says from the side of demand. The country that attracts more consumers also attracts more firms since in this country demand is higher (and as such also profits). The 'cost linkage effect' operates through the costs of production. The country that attracts more industry has a lower price index of manufactures, being that so, for a given nominal wage workers will have propensity to migrate to this location since here they have higher real wages. As such, and considering that real wages do not change in equilibrium, then nominal wages have to fall in the country with more industry, leading at that firms in this country operate with lower marginal costs (and as result also with higher profits).

In the input-output vertical relations case, the 'cost linkage effect' comes from the fact that the country with more intermediate production allows downstream industries to produce at lower costs. This happens to be so, because downstream industries in this country do not have to incur in transport costs to acquire intermediate goods. As consequence, the price index of intermediate goods is lower, and if this is lower, so are the costs of downstream producers in this region since these use intermediates as input. The 'demand linkage effect' arises because the country with more intermediate production also generates a higher demand for firms, because all firms produce goods that others use as intermediate (i.e.: all firms are at same time upstream and downstream producers, or so to say producers and industrial consumers).

In this paper (for both the SCM and RDM) we have two mechanisms of agglomeration. The first is, as in NEG, international labor migration, and the second is phenomena of delocalization. Both mechanisms can affect either the migration decisions of skilled-workers, and the location decisions of firms. Note that we need two mechanisms of agglomeration because location decisions by firms and workers are not so directly connected as in NEG. As we said before, in NEG location decisions of workers and firms are directly connected through the labor market clearing condition, i.e.: labor force and the numbers of firms in a region change by the same proportion[25]. In our model since this is not the case then we need one agglomeration mechanism for workers and another for firms[26].

We show that the RDM besides having a 'demand linkage effect' it has also a 'R&D linkage effect'[27]. The 'demand linkage effect' arises as in the NEG: the country that hosts a higher share of demand promotes firms to choose to locate in this country, since here they make higher profits. The 'R&D linkage effect' arises as the name says as result of R&D investment. R&D allows firms to become

[25] The labor market clearing condition in Krugman (1991) model is $L = n\sigma f$, where L and n and f is like in our model, and σ is the elasticity of substitution. Since f and σ are parameters and therefore are fixed then: $d\ln L = d\ln n$, i.e.: labor force and the numbers of firms in a region change by the same proportion.

[26] Note that even if location decisions by firms and workers are not directly connected, they are however indirectly connected since as it should be expected location patterns of firms and workers affected each other mutually.

[27] The SCM has only demand effects at work in promoting agglomeration.

more competitive than their competitors, since who performs higher levels of R&D investment has lower marginal costs and as consequence the capacity of practising lower prices without affecting (or even increasing) prices. In our model R&D investment happens to be higher where more skilled-workers and demand are located. Therefore we have a kind of 'home market effect' on R&D investment. As consequence, firms will be more competitive in the country with more skilled-workers. 'Home market effects' on R&D result in 'competitiveness effects' that arise therefore as a result of asymmetric labor markets. Both these forces are centripetal, (i.e.: promote the agglomeration of economic activity), in the next section we show the centrifugal forces (i.e.: forces that promote dispersion) present in our model.

4.1 Demand Effect

As we have said above, the RDM also has 'demand linkage effects' as in NEG literature, since local sales also increase with local demand. In fact, the derivative of local sales in relation to u is:

$$\frac{dq}{du} = -t\theta^2 L \frac{\left(\gamma(2N(N+2)(1-s)+1)-\theta^2 M(2N(1-s)+1)\right)}{\phi(N+1)} \tag{25}$$

It can be checked that this derivative is positive[28]. Then as result, when local demand increases local sales also increase. Therefore, as in NEG we can also have cumulative causation effects operating from the side of demand. When the share of skilled-workers in a location increase, demand also increase (since workers are at same time labor force and consumers) and in consequence also the level of local sales (q). Being that so, firms locate in this region have higher profits, and this in turn will promote the emergence of new firms in this location.

4.2 R&D Linkage Effect

The R&D linkage effect in turn, arise as result of two effects: 'home market effects' (for now on HME) in the level of R&D; and 'competitiveness effects'. The first says that firms located in the country with a larger pool of skilled-workers perform higher levels of R&D. The second states that as consequence of these HME in R&D, firms in this country will become more competitive.

4.2.1 Home Market Effects in R&D

To show the existence of HME in R&D, we look at the derivative of R&D investment in relation to local share of skilled-workers:

$$\frac{dk}{du} = -\frac{\theta L t}{\phi}\left(\gamma\left(2\left(1-s\right)N+1\right)-\theta^2 M\right) \tag{26}$$

[28] To see this, note that the multiplicative term is always positive, and the same happens with what is inside parenthesis since $\gamma > \theta^2 M$.

It can be easily seen, that the level of R&D investment always increase with the share of local skilled-workers[29]. Firms located in the region that hosts a higher share of skilled-workers (and demand) will perform higher levels of R&D investment. Local labor markets can therefore influence local industry dynamics, and in this case, R&D investment. As such, as a result of asymmetric labor markets and R&D investment, firms can become endogenously asymmetric.

4.2.2 Competitiveness Effects

We can now see more clearly the 'competitiveness effect' at work by looking at the derivative of exports and local sales in relation to local share of skilled-workers. Note first that under the SCM the derivative of exports in relation to u is zero (since exports do not depend on u, see equation 11), i.e.: labor market spatial patterns do no affect the behavior of exports. On the contrary, under the RDM, exports depends on labor market spatial patterns. Furthermore note that the derivative of exports in relation to u is equal to the derivative of local sales in relation to u, i.e.: equation 25, then:

$$\frac{dx}{du} = \frac{dq}{du} \tag{27}$$

As we have seen equation 25, is unambiguously positive. If it is easy to see why local sales increase with u (if demand increases in a country so also local sales) the same is not the case with the derivative of exports. Explaining why the derivative of exports increase with u can also help us to see that not only 'demand effects' are at work for local sales to increase with u. The rational for exports to increase with u, is that when u increases, k also increases (as a result of 'home market effect' on R&D), and so local firms become more competitive than foreign firms, since R&D reduces marginal costs. As consequence of this 'competitiveness' effect, firms located in the country with more skilled-labor can gain market share in the foreign market, since they are more competitive than foreign firms. Then local sales increase with u, not only because local markets are expanding but also because local firms become more competitive what allows them to gain market share at the expense of less competitive firms (namely foreign firms).

We are now in conditions to explain the 'R&D linkage effect'. If the share of skilled-workers increase in a location then the level of R&D investment performed by local firms also increase (i.e.: HME in R&D). As consequence, firms in this country will have lower marginal costs (C), what make these firms also more competitive (i.e.: competitiveness effect). If this is so, then both local sales and exports increase, what leads to an increase in profits, and as such to the attraction of other firms.

[29] In fact, the derivative of k in relation to u is always positive, since what is inside parenthesis is always positive. To see this, note that the first term is always positive and being that so, it means that the first term is also always bigger than the second since $\gamma > \theta^2 M$.

Therefore, we can have 'cumulative causation effects' that can arise from the side of R&D investment. Starting from a symmetric firm and skilled-worker location equilibrium (i.e.: $s = u = \frac{1}{2}$), if one skilled-worker moves, firms in the recipient region will start to invest more in R&D and to gain competitive edge over firms in the other market. If this happens, other skilled-workers will desire to move to this location so that they can use their skill to match the demand for skilled-labor made up by firms. The same will happen with firms: new firms will be promoted to arise in this region since more skilled-workers will allow them to perform higher levels of R&D investment, and take advantage of new profit opportunities associated with this fact.

As in the NEG there are 'demand' and 'costs linkages effects', in our model we have a 'R&D linkage effect'. R&D through the labor market channel allow firms to become endogenously asymmetric: firms that perform higher levels of R&D investment will have lower marginal costs, and this allows them to make higher profits. Since firms that invest more in R&D are located in the region with more skilled-workers then other firms will follow their example locating in this location. This in turns, attracts more skilled-workers, and since more skilled-workers means the possibility of performing higher levels of R&D investment, the process can feed up in itself (i.e.: cumulative causation) and promote agglomeration. Note however, that this R&D investment linkage effect has both demand and cost effects: it increases demand because reduces the cost of production and as such prices.

5 Dispersion Forces: Competition Effects

Counterbalancing the centripetal forces described above ('demand' and 'cost linkage' effects that pull the economic activity together), NEG considers mainly one centrifugal force that contributes for the dispersion of economic activity: competition. When firms start to agglomerate in one location competition will be fiercer and therefore prices will go down, and as consequence also profits. However, in NEG for certain parameter values, the most likely outcome of the interplay between centripetal and centrifugal forces will be the agglomeration of economic activity and centre-periphery patterns[30].

In our model, as in NEG, the main dispersion force is also competition. This 'competition effect' also arises in the same fashion as in the NEG. As local competition increases prices goes down cannibalizing the profits of local firms.

[30] Namely, in Krugman (1991), agglomeration is promoted for low transport costs, higher share of income spent on manufactures and low elasticity of substitution between varieties (that Krugman interprets as presence of scale economies). Venables (1996), besides these, also points out that agglomeration can be promoted when a higher share of demand by industrial firms is spent on intermediates. According to NEG literature, low elasticity of demand allows exploitation of scale gains by concentrating production in few locations. Demand patterns biased for manufactures supports larger agglomerations of industrial firms. On the same fashion, when demand for intermediate production is important, clusters of vertically linked industries are more easily supported. Finally, low transport costs permits firms to supply markets from central locations.

In fact the derivative of P in relation to s, is:

$$\frac{dP}{ds} = -Nt^{\frac{\gamma(N+1)-2\theta^2 M(1-r)}{\varphi(N+1)}} < 0 \qquad (28)$$

This derivative is unambiguously negative. However, also as in NEG, for certain parameters and variables values, agglomeration can arise. This happens when 'demand' and 'R&D linkage effects' are stronger than the 'competition effect'. Bellow we will analyze when this can be the case.

6 Location Game: Firms

In this section, we solve for the location game of firms by looking at the spatial behavior of profits. We assume a whatever spatial distribution of firms and skilled-workers (i.e.: any s and u), and then check in what conditions agglomeration or dispersion is promoted. By doing this we can learn more about the spatial location dynamics of the high-tech sector.

We define $\Delta\Pi$ as the difference in profits that a representative firm can obtain between being located at home or at foreign (i.e.: the gain in profits from choosing to produce at home instead at foreign), and we assume that location choices by firms are made based on the profit differential between regions.:

$$\Delta\Pi(s,u) = \Pi(s,u) - \Pi^*(s,u) \qquad (29)$$

Then, if $\Delta\Pi = 0$, no location attract more firms than the other (i.e.: delocalization is not promoted). If $\Delta\Pi > 0$, there are economic incentives to the appearance of new firms at home, i.e.: firms delocalize from F to H; if $\Delta\Pi < 0$, there are incentives to the appearance of new firms in the foreign country, i.e.: firms delocalize from H to F. Also note that, the difference in profits depend both on the comparative level of competition (s) and on labor markets and demand (u) at H and at F. Therefore, we are also interested in knowing how competition, demand and labor markets affect the spatial dynamics of firms location decisions.

Delocalization can therefore result in phenomena of regional growth and decline, made possible by comparative local profit opportunities. In this sense, agglomeration is only triggered when firms in a region make continuously more profits than in the other region. We will make use of the labor market equilibrium conditions to prove that in case a region is assisting to regional growth (due to profit opportunities) the other one will assist to regional decline. This, as in NEG, is due to some level of symmetry in our model.

Finally we assume, that the location decision of firms at time T depend on the profit differential in the previous period $(T-1)$, and as such also on the distribution of firms (s) and skilled-workers (u) at $T-1$. This temporal interpretation is needed in order to adjustment (i.e.: migration and delocalization) to occur.

6.1 Standard Cournot Model

We start the analysis of firm spatial behavior in the SCM, by looking at the labor market equilibrium implied by this model. First, note that skilled-labor requirement per firm equals $L_i = c(q+x)+f$, and that $uL = sNL_i$. Then labor market equilibrium implies:

$$sN = \frac{uL}{c\,(q+x)+f} \tag{30}$$

It can be easily seen that the number of home firms (sN) increase with the local share of skilled-labor (u), and total world endowment of skilled-labor (L); and decrease with marginal costs (c), output per firm ($q+x$), and fixed costs (f). The number of firms in a country increases when marginal cost and fixed costs are low, since low levels of these costs represent more profits (in both cases) and lower barriers to entry (in the second case). On the same fashion, if output per firm is high, there is less market for new firms to enter. Also the number of local firms increase with the local share of skilled-workers because it allows more firms to open business there due to higher availability of inputs (i.e.: skilled-labor). Since local labor markets at home and foreign are symmetric (i.e.: if workers increase at home, they decrease in the foreign country) then through the labor market equilibrium condition we know that when firms in one country are increasing in the other they are decreasing. Therefore, agglomeration type of equilibrium can be a result of this model.

We now look at the profit differential in the SCM to see more clearly the spatial behavior of firms:

$$\Delta\Pi = \frac{4Mt}{(N+1)\,b}\left(D\left(r-\frac{1}{2}\right)-\frac{t}{2}\left(N\left(s-\frac{1}{2}\right)+\left(r-\frac{1}{2}\right)\right)\right) \tag{31}$$

The sign of the profit differential will depend on the relative distribution of firms and skilled-workers between locations. However, in the general case is not easy to sign the profit differential equation. We opt by study the behavior of the profit differential around the symmetric equilibrium. Then, the first thing to note is that if both firms and skilled-workers are evenly distributed (i.e.: $s=r=u=\frac{1}{2}$) then no phenomena of regional growth or decline can be triggered because the difference in profits cancel to zero (i.e.: dispersion is the only stable equilibrium). Given this, we look instead at when there is only partial symmetric equilibrium, i.e.: when only skilled-workers are symmetrically located (i.e.: $r=u=\frac{1}{2}$), or alternatively when firms are symmetrically located (i.e.: $s=\frac{1}{2}$).

The idea behind this strategy is the following. Take the example of when we set $s = \frac{1}{2}$. We must think what will happen to this location pattern initially assumed, i.e.: we study whatever agglomeration or dispersion is promoted in subsequent in subsequent periods of the game. This can give us some clues of how and based in what, firms make their location choices.

6.1.1 Symmetric firm location equilibrium

We start first by setting $s = \frac{1}{2}$ (i.e.: symmetric firm location equilibrium). In this case the profit differential becomes:

$$(\Delta\Pi)_{s=\frac{1}{2}} = \frac{4Mt}{(N+1)\,b}\left(r - \frac{1}{2}\right)\left(D - \frac{t}{2}\right) \tag{32}$$

As long the OMC for the SCM is satisfied, the difference in profits is positive for $r > \frac{1}{2}$ (i.e.: $u > \frac{1}{2}$), negative for $r < \frac{1}{2}$ (i.e.: $u < \frac{1}{2}$), and zero for $r = \frac{1}{2}$ (i.e.: $u = \frac{1}{2}$). Therefore, the region that hosts a large share of skilled-workers will assist to phenomena of regional industrial growth that can lead to agglomeration.

We can take now a temporal interpretation to our model. If at $T = 0$, $s = r = u = 1/2$, then if in subsequent periods there is no perturbation in the spatial location of firms or workers, no phenomena of regional growth or decline will be promoted. Conversely, if at $T = 0$, $s = 1/2$, but $u > 1/2$, then in the subsequent periods firms will delocalize from foreign to home.

6.1.2 Symmetric skilled-workers location equilibrium

Now we set $u = \frac{1}{2}$, i.e.: skilled-workers symmetric location equilibrium. The difference in profits in this case is simply:

$$(\Delta\Pi)_{u=\frac{1}{2}} = -\frac{2MNt^2}{(N+1)\,b}\left(s - \frac{1}{2}\right) \tag{33}$$

In terms of location patterns, when there is a symmetric distribution of skilled-workers, the symmetric spatial equilibrium of firms (i.e.: $s = \frac{1}{2}$) is also the only stable equilibrium, i.e.: in any period $T = 0, 1, ...\Upsilon$, $r = u = s = 1/2$, is the only stable equilibrium whatever the initial distribution of firms, or any perturbation in this equilibrium. To see this, note that for $s > \frac{1}{2}$ the difference in profits is always negative and as such firms will delocalize from H to F until the symmetric firm location equilibrium is restored. For $s < \frac{1}{2}$ the difference in profits is always positive and as such firm will delocalize from F to H until once again the symmetric equilibrium is restored. For $s = \frac{1}{2}$ the difference in profits is always null and therefore no spatial dynamic phenomena is triggered.

6.2 R&D Investment Model

We start again by analyzing the labor market equilibrium condition, since this can be informative about the behavior of spatial industrial patterns. Note first, that per firm skilled-labor requirement is $L_i = (c - \theta k)(q + x) + \Gamma$, with $\Gamma = \gamma\frac{k^2}{2} + f$, and that $uL = sNL_i$. Then, through the labor market equilibrium condition we get:

$$sN = \frac{uL}{(c - \theta k)(q + x) + \Gamma} \tag{34}$$

It can be easily seen that the number of home firms (sN) increase with the cost-reducing effect of R&D (θ), and the local share of skilled-labor (u); while it decreases with initial marginal costs (c), fixed costs (Γ), output per firm ($q+x$), and the cost of R&D investment (γ). The number of local firms decrease with marginal costs and fixed costs since both affect negatived profitability and the fixed costs also deter entry. The number of firms also decrease with total outputs: when output per firm is high, potential entrants find it harder to enter to find a niche in the market. The number of firms increase with the cost-reducing effect of capital: when R&D is very effective firms find it easy to stay and compete in the market place. On the contrary it decrease with the cost of R&D: higher costs of R&D turns R&D not so effective to face competition. Finally, local firms increase with the share of skilled-labor, since this means greater availability of inputs.

The effect of R&D investment (k) by itself is not straight forward. From one side, it can increase the number of local firms since it reduces marginal costs, but since it also increases fixed costs it can make it more difficult for a firm to enter the market. The net effect of R&D will the depend on the balance between this two. Substituting for $\Gamma = \gamma\frac{k^2}{2} + f$, and rearranging the equation 34 we get:

$$sN = \frac{uL}{c(q + x) + f - k\left(\theta(q + x) - \gamma\frac{k}{2}\right)}$$

Then, the number of local firms increase with R&D investment, if and only if:

$$\theta(q + x) > \gamma\frac{k}{2} \tag{35}$$

If the level of output (local sales more exports) is bigger than half of the R&D weight by the relation between the cost of R&D and the cost-reducing effect of R&D, then when k increases in one location, the number of local firms (sN) also increases. The number of firms in one location can increase with R&D because firms prefer to be in the country where higher levels of R&D

investment is performed, since R&D can make them more competitive than firms in the other country. Note however, that this only happens if the above restriction is satisfied.

We know that due to 'demand effects' when u increases sN also increases, but since when u increases, k also increases (due to HME in R&D), then if 35 is satisfied, sN can increase by a larger proportion than the one resulting only from 'demand effects', i.e.: R&D can reinforce Krugman (1980) 'home market effects': countries with a higher share of demand have a proportionally higher share of industry, but now not only due to 'demand effects' but also to R&D investment.

Since R&D through the labor market channel can make firms more competitive in one location, the less competitive firms (i.e.: firms in the other location) will be tempted to delocalize, since if they do not do so it will become harder for them to face competition (because the share of skilled-labor and the level of R&D investment per firm are decreasing in this country). As such when one country assist to industrial growth the other can assist to industrial decline.

Now we would like to analyze the static behavior of the profit differential equation in the RDM. However the expression for profit differential is too cumbersome to be presented. Therefore we follow the strategy to fix at the time s and u to be equal to one-half, and then based in this two defined spatial patterns analyze the behavior of the profit differential.

6.2.1 Symmetric firm location equilibrium: Strategic Location

We start first by setting $s = \frac{1}{2}$ (i.e.: symmetric firm location equilibrium). In this case the profit differential becomes:

$$(\Delta\Pi)_{s=\frac{1}{2}} = -\frac{2M\gamma t}{\phi}\left(r - \frac{1}{2}\right)\left(D - \frac{t}{2}\right)\left(2\gamma - \theta^2 M\right) \tag{36}$$

As long as, the OMC for the SCM is satisfied and $\gamma > \theta^2 M$, the difference in profits is positive for $r > \frac{1}{2}$ (i.e.: $u > \frac{1}{2}$), negative for $r < \frac{1}{2}$, and zero for $r = \frac{1}{2}$[31]. The region that hosts a large share of skilled-workers will assist to phenomena of regional industrial growth.

This is the same behavior as in the SCM. Then, once again, the temporal evolution is similar. The symmetric equilibrium ($s = r = u = 1/2$) will tend to be break if one region starts to host more skilled-workers. Agglomeration in this case is promoted depending on the spatial location decisions of skilled-workers.

But while in the SCM location of firms were promoted in the country with more skilled-labor due to demand reasons, in the RDM this is due also to strategic motives (i.e.: firms want to locate in the country with more skilled-labor to perform higher levels of R&D in order to be more competitive). Therefore in the R&D model we have strategic location choices.

[31] To see this, note that the multiplicative term is always positive (since $\phi < 0$), and that what is inside the third parenthesis is also always positive since $\gamma > \theta^2 M$.

6.2.2 Symmetric skilled-workers location equilibrium: Symmetric Firms

Now we set $u = \frac{1}{2}$, i.e.: worker symmetric location equilibrium. The difference in profits in this case is simply:

$$(\Delta\Pi)_{u=\frac{1}{2}} = -\frac{2MNt^2}{N+1}\left(s - \frac{1}{2}\right) \tag{37}$$

This is exactly the same expression as in the SCM (see equation 33). Then the symmetric equilibrium ($s = \frac{1}{2}$) is the only stable equilibrium. This happens to be so, because when workers are symmetrically distributed between countries firms, in whatever country they are located, perform the same levels of R&D investment and as such we fall into the SCM case where all firm have the same level of marginal costs. In fact when skilled-workers are symmetric located, the R&D parameters (γ and θ) do not affect the difference in profits between locations:

$$\left(\frac{d\Delta\Pi}{d\gamma}\right)_{u=\frac{1}{2}} = \left(\frac{d\Delta\Pi}{d\theta}\right)_{u=\frac{1}{2}} = 0$$

Therefore, when the share of skilled-workers is fixed at the symmetric level, the difference in profits is not affected by changes in the R&D investment parameters (γ, and θ). This shows once again, that R&D investment only plays a role in regional luck when regions differ at the level of labor markets. This is so, because when skilled-workers are evenly distributed, R&D investment by firms do not turn them asymmetric, i.e.: if they invest the same, they have the same marginal costs, and as such none of the firms is more competitive than the others. Therefore, the RDM is only interesting (and only differs from the SCM) when it allows firms to become asymmetric through asymmetries in the labor markets and consequent asymmetries in the levels of R&D investment (i.e.: when it allows u to not be fixed at one-half).

7 Location Game: Skilled-Workers

In this section, we analyze the migration and location decision of skilled-workers. As in Ottaviano et al. (2001) workers migrate in response to indirect utility differentials, i.e.: workers migrate to the region where they can get a higher utility. Therefore migration decisions by workers is given by:

$$\Delta V(s,u) = V(s,u) - V^*(s,u) \tag{38}$$

Where ΔV is the indirect utility differential between living at home and foreign (i.e.: the gain in the indirect utility for a representative consumer from choosing to live at home instead at foreign), V indirect utility in the home country, and V^* indirect utility in the foreign country. Note that these relations depend on both the share of skilled-workers (u) and also share of firms (s) in a location. Therefore, *a priori* we should expect that the indirect utility differential is affected by both the levels of competition (i.e.: s) and labor markets and demand (i.e.: u) in a region.

Once again we derive ΔV in relation to s and u because the game can be interpreted in a temporal way, i.e.: the spatial economic pattern at $T-1$ can influence the spatial economic pattern at T and subsequent periods. Therefore, once again the location decisions of workers at period T, is made by taking in consideration the indirect utility differential, and as such the distribution of firms (s) and skilled-workers (u) at $T-1$. Conversely then, if $\Delta V = 0$, workers have no incentives to move; if $\Delta V > 0$, workers move to H; if $\Delta V < 0$, workers move to F[32].

While in the profit differential analysis is more or less obvious in what ways demand and competition affects location decisions of firms (i.e.: demand works in favor of agglomeration since it increases profits, while competition against it since it decreases profits), in the case of the indirect utility differential this is not so straight-forward and needs further qualification. Competition is in principle favor to the agglomeration of skilled-workers both because price levels must be lower in the country with more firms (and therefore utility is higher), but also because skilled-workers can more easily find a job on what they like to do (i.e.: in the case of the RDM to use their skill to perform R&D investment). In this sense the spatial behavior of skilled-workers differs from the spatial behavior of firms: firms prefer location with lower levels of competition while skilled workers the contrary. This is one example why we should separate workers and firm location decisions since they can have different motivations in what concerns the choice of location.

The effect of the distribution of demand and labor markets on the indirect utility of skilled-workers is not so easily evaluate. In fact the demand effect in the SCM has no influence in the indirect utility of consumers (remember that the price levels are independent of the distribution of demand or labor markets). However in the RDM this is not exactly the case, not only because price levels depend on the distribution of demand, but and most importantly because firms located in the region that hosts a large share of skilled-workers perform higher levels of R&D than their counterparts in the other region, making firms in this region more competitive. This means lower marginal costs and lower prices and as consequence higher utility, but also a higher demand for skilled-labor (since firms need more skilled-labor to perform R&D investment).

7.1 Standard Cournot Model

In the SCM the indirect utility differential is:

[32] Note that as we had mentioned before, since wages are fixed (i.e.: income is exogenous) the migration decision by workers/consumers is driven only by differences in price levels.

$$\Delta V = \frac{2N^2 t}{b\,(N+1)^2}\left(s - \frac{1}{2}\right)\left(D - \frac{t}{2}\right) \tag{39}$$

Note that ΔV does not depend on the distribution of demand and labor markets (i.e.: u). Therefore, skilled-workers do not really care where other skilled-workers are located, they only care where firms are located. Furthermore note that for $s = \frac{1}{2}$ (symmetric firm location equilibrium), $\Delta V = 0$. This means that when firms are equally distributed workers do not gain in moving. However, for $s \neq \frac{1}{2}$, ΔV has always the same sign as $\left(s - \frac{1}{2}\right)$ as long as the OMC for the SCM is satisfied. If $s > \frac{1}{2}$ (i.e.: home host a bigger share of firms) consumers want to be located at home; if $s < \frac{1}{2}$ consumers move to foreign. Therefore, we have agglomeration of skilled-workers if market size is sufficiently large comparatively to trade costs, and when a region hosts a larger share of industry.

In a temporal evolution, we see that skilled-workers will never move if firms are symmetrically located. However, it is only needed that one additional firm delocalize to a country so that workers start to move to this location. Agglomeration can result in subsequent periods, due to migration of skilled-workers.

7.2 R&D Investment Model

In the RDM the indirect utility differential is:

$$\Delta V = \frac{2N^2 t}{\varphi(N+1)^2}\left(s - \frac{1}{2}\right)\left(\gamma\,(N+1)\left(D - \frac{t}{2}\right) + 2\theta^2 M t\left(s - \frac{1}{2}\right)\left(r - \frac{1}{2}\right)\right) \tag{40}$$

Now (and contrary to the SCM), ΔV depends on the distribution of demand and labor markets spatial patterns, i.e.: skilled-workers also care where the other workers are located. As in the SCM, $\Delta V = 0$ for $s = \frac{1}{2}$. This means, that when firms are symmetrically distributed between locations, then even if workers are unsymmetrical distributed, skilled-workers have no incentives to migrate. Also for $s \neq \frac{1}{2}$, ΔV has always the same sign as $\left(s - \frac{1}{2}\right)$ as long as the OMC for the SCM is satisfied and $\varphi > 0$ (i.e.: $\gamma > \theta^2 M$, or so to say R&D investment is costly and/or the cost reducing effect of investment is not too high). The proof follows in the appendix. Then, the region that hosts more firms attracts more skilled-workers.

Taking a temporal evolution, if one country by any reason attracts more industry, skilled-workers in subsequent periods of the game will migrate to this location.

7.2.1 Comparative Statics: Price and Job-Match Effects

We look now at the behavior of the indirect utility differential. We start with the derivative of the indirect utility differential in relation to s:

$$\frac{d\Delta V}{ds} = \frac{2N^2 t}{\varphi (N+1)^2} \left(\gamma (N+1) \left(D - \frac{t}{2} \right) + 4\theta^2 M t \left(s - \frac{1}{2} \right) \left(r - \frac{1}{2} \right) \right)$$

As shown in the appendix ΔV increases with s (i.e.: skilled-workers prefer to locate where more firms are located), as long as, the OMC for the SCM is satisfied and $\gamma > \theta^2 M$. This is so for two reasons. First, because when s increases, P decreases and being that so, V also increases. Higher competition results then in 'price effects': workers prefer to live in countries where they can get the hi-tech good at lower prices, and this is so in the country with more industry because competition leads prices down. Second, also, if s increases this means more jobs for skilled-workers. Therefore we also have 'job-match effects': skilled-workers prefer to live in the country that allows them to use their skill.

Looking now at the derivative of the indirect utility differential in relation to local share of skilled-workers:

$$\frac{d\Delta V}{du} = \frac{4\theta^2 L N^2 t^2}{\varphi (N+1)^2} \left(s - \frac{1}{2} \right)^2$$

It turns out that the derivative of the indirect utility differential in relation to u is positive as long as $\gamma > \theta^2 M$. This arises, because when u is increasing the level of R&D investment is also increasing and as such firms in this location become more competitive than firms in the other location, and practice more appealing price policies, that leads to higher levels of consumer utility in this region (i.e.: V increases). Therefore, we have again 'price effects', but now resulting from R&D investment: skilled-workers prefer locations with lower prices in the hi-tech sector, and this is so in the country where firms perform more R&D.

Furthermore, when this happens (i.e.: when u increases, and k as consequence also increases), more firms delocalize to this location (as long as equation 35 is satisfied) leading to more job opportunities in the hi-tech sector. Once again we can have 'job-match-effects', but now the increase in the number of jobs for skilled-workers, that happens when the number of firms in a location increase, is feed by the R&D channel. Therefore, skilled-workers prefer locations where other skilled-workers are located, both because utility is higher and it is also easy to find a job in what they like to do (i.e.: R&D activities). This comes in deep contrast with the SCM where migration dynamics were not influenced by u.

8 Stability of the Agglomerated Equilibrium

As in NEG, given the symmetry in the base model, we have three 'natural' spatial equilibriums: the symmetric equilibrium (with $s = u = \frac{1}{2}$), agglomeration at home ($s = u = 1$), and agglomeration at foreign ($s = u = 0$). In this section we study the stability of the agglomerated equilibrium. In the next section we illustrate graphically the spatial equilibriums implied by our model.

To analyze in what conditions the propensity for agglomeration is stable, i.e.: in what ways the agglomerated equilibrium survives to short-term phenomena of delocalization and migration, we look at both the indirect utility differential and at the profit differential at $s = u = 1$: the former drives migration decisions by skilled workers, the later determines what locations are more attractive for firms. If $(\Delta\Pi)_{s=u=1}$ (or $(\Delta V)_{s=u=1}$) is equal or bigger than zero then the agglomerated equilibrium is stable. If $(\Delta\Pi)_{s=u=1}$ (or $(\Delta V)_{s=u=1}$) is smaller than zero then the agglomerated equilibrium is unstable.

8.1 Standard Cournot Model

In this sub-section, we study under what conditions the agglomerated equilibrium in the SCM is stable for firms and skilled workers (i.e.: it is more advantageous agglomeration than dispersion).

8.1.1 Profit Differential

Setting $s = u = 1$ in the expression for profit differential between locations we get:

$$(\Delta\Pi)_{s=u=1} = \left(DL - \frac{t}{2}(NM + L)\right)\frac{2t}{(N+1)\,b} \tag{41}$$

This expression is always positive as long as:

$$D > \frac{t}{2}\left(\frac{NA}{L} + (N+1)\right) \tag{42}$$

This condition is satisfied (and therefore the agglomerated equilibrium is stable) for higher values of D and L; and for lower values of t, A and N. Dispersion is promoted when the contrary happens. A large market size makes exports more profitable and therefore agglomeration more attractive. Lower values of trade costs allow firms to serve distant markets from central locations. Very competitive world markets (high N) supports more symmetric equilibriums because local competition is very fierce under agglomeration. A large world endowment of skilled-labor favors agglomeration because firms can more easily find locally inputs need to production. In the same way, a large world endowment of unskilled labor discourages agglomeration because the part of world demand not involved in production in the HTS is big, making greater the advantages of exporting and smaller the advantages of producing locally since this factor is not used in production.

8.1.2 Indirect Utility Differential

In the SCM, u does not appear in the expression for the indirect utility differential. Then, we just need to make $s = 1$, to obtain:

$$(\Delta V)_{s=1} = \frac{N^2 t}{b\,(N+1)^2}\left(D - \frac{t}{2}\right) \tag{43}$$

This expression is always positive as long as the OMC for the SCM is satisfied. When such happens all consumers want to move to this location. This means that when firms are agglomerated the only stable equilibrium for skilled-workers is also the one where all consumers are also agglomerate ($u = 1$). Trade therefore, not only promotes the agglomeration of firms but also of skilled-workers. Then, when countries are more interdependent (as is the case when trade relations are established) agglomeration of manufacturing activities is more likely. On the contrary when trade is not possible or restricted agglomeration is not stable for skilled-workers.

8.2 R&D Investment Model

In this sub-section, we study if the agglomerated equilibrium in the RDM is stable for firms and skilled workers.

8.2.1 Profit Differential

Setting $s = u = 1$ in the expression for the profit differential of the RDM gives a quite cumbersome expression. Also, the same happens for establishing conditions for the agglomerated equilibrium to be stable. Given these difficulties, we choose instead to set $s = r = 1$ in the profit differential expression of the RDM. This is equivalent to assume that all workers (skilled plus unskilled) are mobile. By doing this, we are able to show formally (even if not totally correct) an idea of the stability of the agglomerated equilibrium for firms.

Then, at $s = r = 1$ the profit differential simplifies to:

$$(\Delta\Pi)_{s=r=1} = tM\gamma\frac{\theta^4 M^2\left(D-\frac{t}{2}\right)+2\gamma^2\left(D-\frac{t}{2}(N+1)\right)-\gamma\theta^2 M\left(3D-\frac{t}{2}(N+3)\right)}{(\theta^2 M-\gamma)\phi} \tag{44}$$

Since the denominator is always positive, then this expression is also always positive (i.e.: the agglomerated equilibrium is stable) if:

$$D > \frac{1}{2}\left(\gamma\,(N+1) - \theta^2 M\right)\frac{t}{\gamma - \theta^2 M} \tag{45}$$

The relation on the right side of this equation is always positive, as long as $\gamma > \theta^2 M$. Then, the first thing to note is that for higher values of market size (D) the agglomerated equilibrium tends to be stable (since the condition above is satisfied). On the contrary dispersion is promoted for higher values of N, t, M, and θ; and for lower levels of γ[33].

Higher levels of competition (high N) makes agglomeration less likely, since firms are more hurt when they locate together. High trade costs (t) and low market size (D) makes exports less profitable and as such firms prefer to be close to local markets. A high number of potential world consumers (high M) makes exports more profitable since markets are bigger, and as such agglomeration more attractive. High values of cost-reducing effect of R&D investment (θ) makes competition fiercer and as consequence firms prefer dispersion instead of concentration. On the contrary, low costs of R&D investment (γ) can help less competitive firms to face competition and counterbalance agglomeration forces.

8.2.2 Indirect Utility Differential

Setting both s and u equal to one in the indirect utility differential we obtain:

$$(\Delta V)_{s=u=1} = \frac{tN^2}{2\varphi\left(N+1\right)^2}\left(2\gamma\left(N+1\right)\left(D-\frac{t}{2}\right)+\theta^2 Lt\right) \tag{46}$$

This expression is always positive, as long as, the OMC for the SCM is satisfied (since then the expression in parenthesis is positive), and $\gamma > \theta^2 M$ (since then the multiplicative term is also positive). If this is so, then the agglomerated equilibrium is stable because workers have no incentives to migrate to other location. In order to see more clearly how the agglomerated equilibrium interacts with the model structural parameters and variables note that ΔV at $s = u = 1$ is positive if:

$$D > \frac{1}{2}t\frac{\gamma\left(N+1\right)-\theta^2 L}{\gamma\left(N+1\right)} \tag{47}$$

It can be shown that the agglomerated equilibrium is promoted for higher values of D, L, and θ; and for lower values of t, N, and γ[34]. Then, the behavior of D, L, t, and N is similar for firms and skilled-workers (i.e.. agglomeration for firms and skilled-workers is promoted in a similar fashion in what concerns D, L, t, and N). However, this behavior differs in what concerns the R&D parameters. While skilled-workers prefer agglomeration when the cost-reducing effects of R&D is high and the cost of R&D investment is low; for firm is exactly the contrary. Skilled-workers benefit from high θ, and low γ because this make prices lower, while firms prefer low θ, and high γ for competition to not be so harmful.

[33] In fact the derivative of the expression in the right side of this equation is increases with N, θ, t and M, but decreases with γ.

[34] The derivative of the expression in the right side of this equation is increases with N, γ, t and decreases with θ, and L.

9 Spatial Equilibrium

In this section we illustrate graphically the spatial equilibriums implied by our model. For doing this we will use simulation techniques. We attribute values to parameters and variables (more concretely to N, A, L, t, D, and b in the SCM; and N, A, L, t, θ, D, and γ in the RDM). Then we plot the contours of the indirect utility differential and profit differential associated with these values to see migration phenomena by skilled-workers, and plant 'births' and 'deaths' by firms. Next, we join the two curves to evaluate the spatial equilibriums associated. We start with a central case and then we check the sensibility of results as we depart from the central case.

9.1 Standard Cournot Model

The central case in the SCM is: $N = 100$, $A = 50$, $L = 50$ (i.e.: $M = 100$), $t = 2$, $D = 5000$, $b = 1$ (to normalize it to the same intercept as in the RDM). We first show the isoline for the indirect utility differential:

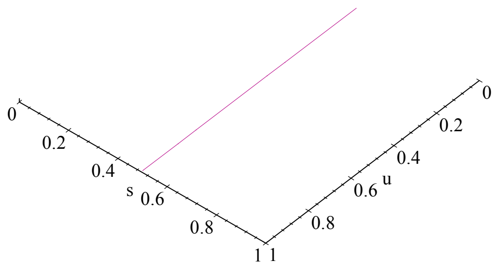

Figure 1: Contours ΔV SCM (central case)

It comes out that to the right of the isoline $\Delta V > 0$, while the contrary happens for the left of the isoline (i.e.: $\Delta V < 0$). This means that workers move to the region with more firms, if we are in the right side of the isoline workers move to home, and the contrary happens in the left side of the isoline.

The isoline for the profit differential is:

In the case of the isoline of the profit differential, the profit differential is positive for the left of the isoline, and negative on the right side of the isoline. Then firms prefer locations with more skilled-workers. As such when we are in the left side of the isoline firms move to home, and the contrary on the right side of the isoline.

Now we can study both the spatial equilibrium of the SCM as well as the stability of the equilibriums found. To do this we plot the two curves together:

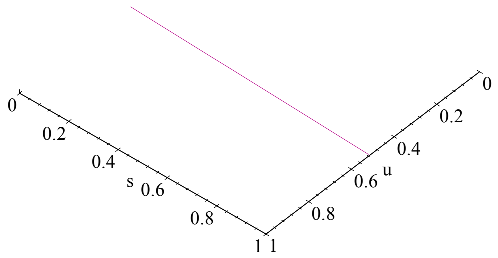

Figure 2: Contours $\Delta\Pi$ SCM (central case)

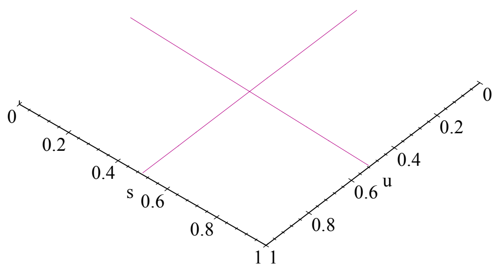

Figure 3: Spatial equilibrium SCM (central case)

We can see that we have three types of saddle-path stable equilibriums, one with both s and u equal to one-half, other with s and u equal to one, and another one with s and u equal to zero). The first equilibrium ($s = u = \frac{1}{2}$) is stable for $s < \frac{1}{2}$ and $u > \frac{1}{2}$ (upper left quarter); and/or $s > \frac{1}{2}$ and $u < \frac{1}{2}$ (lower right quarter). The equilibrium $s = 1$, $u = 1$ (i.e.: agglomeration at home) is stable for $s > \frac{1}{2}$ and $u > \frac{1}{2}$ (lower left quarter). Finally, the equilibrium $s = 0$, $u = 0$ (i.e.: agglomeration at foreign) is stable for $s < \frac{1}{2}$ and $u < \frac{1}{2}$ (upper right quarter).

Sensibility Analysis The first thing to note is that the indirect utility differential is not sensible to different parameter values. The same does not happen with the profit differential. However some common features can be defined when we change the values of the central simulation. First, profit differential results are not sensible to M, increasing or decreasing the total number of workers (skilled plus unskilled) does not affect the results of the central case. Second, decreasing the number of firms, increasing the number of skilled-workers (or decreasing the number of unskilled-workers), decreasing trade costs or increasing market size has also no effects in the results of the central case. Only increasing N, decreasing L (or increasing A), increasing t, and decreasing D, the results are affected. For sufficiently small changes of these variables and parameters[35], the only thing that changes is the inclination of the isoline of the profit differential, as shown in the figure bellow. This means that agglomeration happens for high values of u (i.e.: the agglomerated equilibrium is more difficult to achieve)[36]:

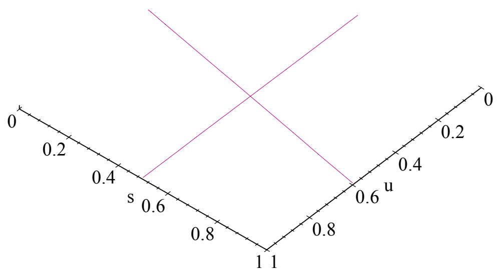

Figure 4: Spatial equilibrium SCM: intermediate N, t, L, A, D

For sufficiently big changes on the values of N, t and D, or A (relatively to L)[37], however the spatial equilibrium of the model is significantly changed. In fact increasing N, t and A above a threshold level, and decreasing D and L bellow a threshold level the agglomerated equilibrium stops to be stable. The only stable equilibrium is the symmetric one. However we have several unstable equilibriums where a region hosts more firms than the other region[38]:

[35] For example for $N = 500$; $A = 90$ ($L = 10$); $t = 8$; $D = 300$.

[36] This figure is plotted for $N = 500$, $A = 50$, $L = 50$ (i.e.: $M = 100$), $t = 2$, $D = 5000$, $b = 1$. A similar figure is obtained, for example, for $A = 90$ ($L = 10$); $t = 8$; $D = 300$.

[37] For example for $N = 10000$, $D = 100$, $t = 100$, or $A = 98$ ($L = 2$). Note that these values of N, D and t imply that trade is not possible (i.e.: regions are in autarchy), since the OMC is not satisfied (i.e.: $D < t\,(N + 1)$). The same does not happen with the values of A (and L), but the condition for agglomeration to be stable (i.e.: $D > \frac{t}{2}\left(\frac{NA}{L} + (N+1)\right)$) is not satisfied. This condition is also not satisfied for the values of N, D and t above mentioned.

[38] This figure is plotted for $N = 100$, $A = 50$, $L = 50$ (i.e.: $M = 100$), $t = 20$, $D = 5000$, $b = 1$. A similar figure is obtained, for example, for $N = 10000$; $D = 100$; or $A = 98$ ($L = 2$).

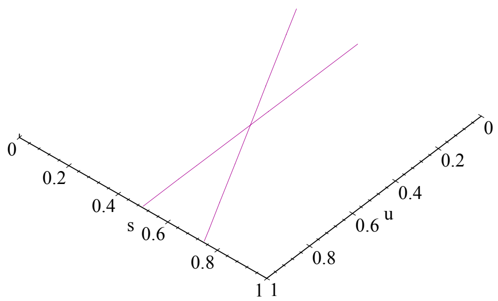

Figure 5: Spatial equilibrium SCM: high N, high t, low D, high A (low L)

9.2 R&D Investment Model

The central case in the RDM is: $N = 100$, $A = 50$, $L = 50$ (i.e.: $M = 100$), $t = 2$, $\theta = 5$, $D = 5000$, and $\gamma = 9000$ (since we need that at least $\gamma > \theta^2 M$, i.e.: $\gamma > 2500$).

We first show the isoline for the indirect utility differential:

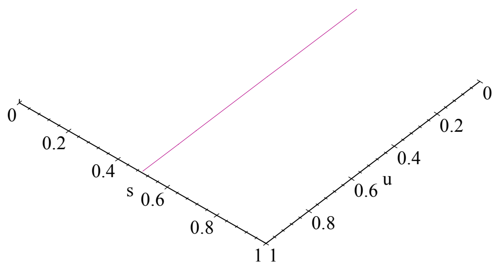

Figure 6: Contours ΔV RDM (central case)

It can be shown that for the left of the isoline $\Delta V < 0$, and the contrary to the right of the isoline (i.e.: $\Delta V > 0$). This means that skilled-workers tend to move to the country with more firms, and in the limit to the country with a share of firms $s = 1$.

The isoline for the profit differential is:

Now for the right of the isoline $\Delta\Pi < 0$, for the left of the isoline we have $\Delta\Pi > 0$. Then, firms will prefer regions with more skilled-labor, and in the

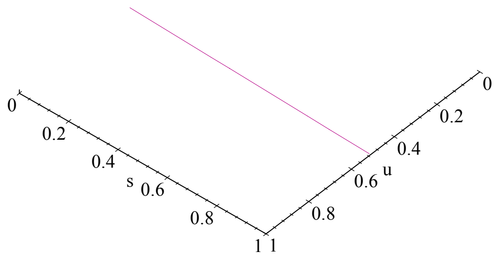

Figure 7: Contours $\Delta\Pi$ RDM (central case)

limit regions with a share of skilled-workers $u = 1$.

We can now study the spatial equilibrium and stability. To do this we just plot together the two curves:

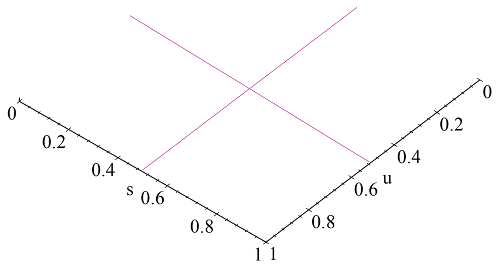

Figure 8: Spatial equilibrium RDM (central case)

We can see that we have three types of saddle-path stable equilibriums, one with both s and u equal to one-half, other with s and u equal to one, and another one with s and u equal to zero. The equilibrium $s = u = \frac{1}{2}$, is stable for $s < \frac{1}{2}$ and $u > \frac{1}{2}$ (upper left quarter); and/or $s > \frac{1}{2}$ and $u < \frac{1}{2}$ (lower right quarter). The equilibrium $s = 1$, $u = 1$ (i.e.: agglomeration at home) is stable for $s > \frac{1}{2}$ and $u > \frac{1}{2}$ (lower left quarter). Finally, the equilibrium $s = 0$, $u = 0$ (i.e.: agglomeration at foreign) is stable for $s < \frac{1}{2}$ and $u < \frac{1}{2}$ (upper right quarter). This behavior is similar to the SCM.

Sensibility Analysis As in the SCM the indirect utility differential is not sensible to different parameter values. The same does not happen with the profit differential. Also, as in the SCM, the sensibility analysis for the profit differential shows some common features for different changes in the values of parameters and variables. In fact decreasing the number of firms, decreasing trade costs, increasing market size, or increasing skilled-labor (but decreasing unskilled-labor in a way such that maintains constant the total number of workers M, skilled plus unskilled) has no effects in the results. Only if we increase N, increase t, decrease D, and decrease γ[39], the results from the central case are changed. However, for sufficiently small changes in the values of these parameters and variables[40] the only thing that changes is the inclination of the isoline of the profit differential (see figure below). This means that agglomeration happens for higher values of u[41], and as such that agglomeration is harder to be achieved:

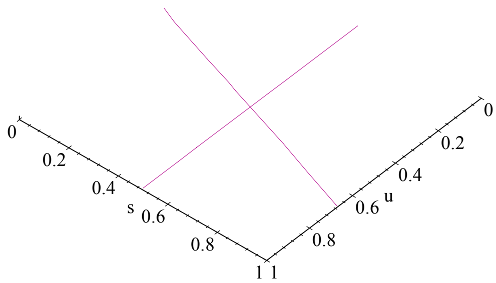

Figure 9: Spatial equilibrium RDM: intermediate N, t, D, γ

For sufficiently big changes on the values of N, t and D[42], however the spatial equilibrium of the model is significantly changed. In fact increasing N, and t above a threshold level, and decreasing D bellow a threshold level the agglomerated equilibrium stops to be stable. The only stable equilibrium is the symmetric one. However we have several unstable equilibriums where a region hosts more firms than the other region (see figure 10)[43].

[39] Note that checking the sensibility of results to γ is the same as checking the sensibility of results to the parameters θ and $M = A + L$ since these parameters are connected through the determinant condition (equation 17). Then decreasing γ is the same as increasing θ or M.

[40] For example for $N = 1000$, $A = 90$ ($L = 10$), $t = 10$, $D = 500$, or $\gamma = 2580$.

[41] This figure is plotted for $N = 1000$, $A = 50$, $L = 50$, $t = 2$, $\theta = 5$, $D = 5000$, and $\gamma = 9000$. A similar figure is obtained, for example, for $A = 90$ ($L = 10$); $t = 10$; $D = 500$; $\gamma = 2580$.

[42] For example for $N = 10000$, $D = 50$, $t = 100$. Note that these values of N, D and t imply that trade is not possible (i.e.: regions are autarchy), since the OMC for the R&D model is not satisfied.

[43] This figure is plotted for $N = 100$, $A = 50$, $L = 50$, $t = 100$, $\theta = 5$, $D = 5000$, and $\gamma = 9000$. A similar figure is obtained, for example, for $N = 10000$; or $D = 50$.

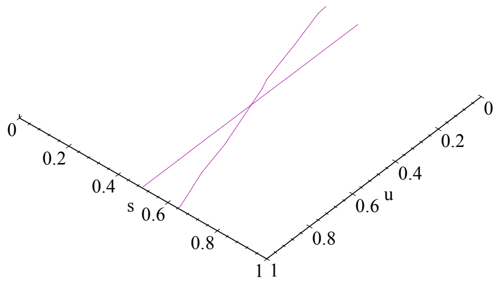

Figure 10: Spatial equilibrium RDM: high N, high t, low D

The behavior of γ is a little bit different of N, t and D. In fact, for very low values of γ (or very high values of θ), the profit differential spatial behavior is as depicted in figure 11[44].

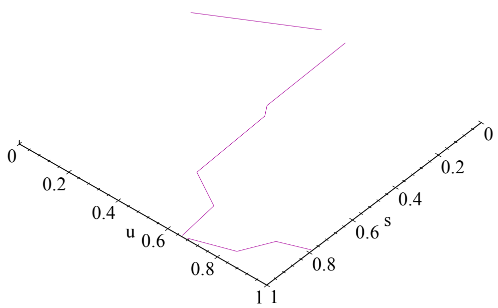

Figure 11: Contours $\Delta\Pi$: low γ, high θ

As before the profit differential is positive for the left of the isoline and negative for the right. However, for the left of the isoline for high values of u and s the profit differential becomes negative; for the right of the isoline the profit differential becomes positive for very low values of s and u. Then, the spatial equilibrium comes as in figure 12.

Now it can be easily seen, that for very high values of u and s; and for very

[44] This figure is plotted for $N = 100$, $A = 50$, $L = 50$, $t = 2$, $\theta = 5$, $D = 5000$, and $\gamma = 2515$. A similar picture is obtained for $\theta = 9.48$. Note that these values of γ and θ, imply that trade is not possible (i.e.: regions are autarchy), since the OMC for the R&D model is not satisfied.

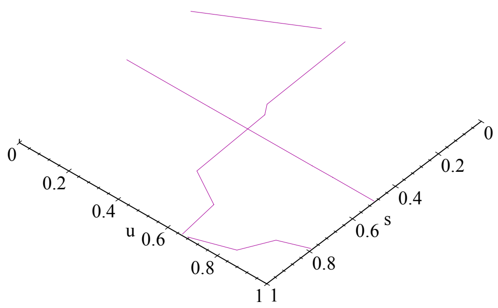

Figure 12: Spatial equilibrium RDM: low γ, high θ

low values of u and s, total agglomeration is not possible. Then as consequence we have that the only saddle-path stable equilibrium is the symmetric one (i.e.: $s = u = \frac{1}{2}$). However, we have multiple equilibrium that are not stable, but none of these are the agglomerated one. This differs from figure 10 where the symmetric equilibrium only stable equilibrium, while here the symmetric equilibrium is only saddle-path stable. This also gives as an additional information about the interconnections between firms and skilled-workers location decisions. In fact we have seen before that while firms prefer agglomeration for high γ and low θ, the contrary happens for skilled-labor. However when both skilled-workers and firms make their location choices, firms preferences are imposed to skilled-workers, i.e.: for low γ and high θ skilled-workers prefer agglomeration but even so they end up choosing dispersion because firms do so. This indicates once again that location choices of skilled-labor are mainly driven by firms location choices.

This shows that R&D investment is a good tool to promote more symmetrical equilibriums either by reducing the costs of R&D or by increasing the effectiveness of R&D. Therefore, the policy questions for lagged regions is how to lower the costs of R&D investment, or to increase the efficiency of R&D investment for local firms, in order to counter-balance centripetal forces.

10 Discussion

We had study the influence of R&D investment on location decisions of firms and skilled-workers. Contrary to most literature on the NEG, based on the Dixit-Stiglitz monopolistic competition model, we had used instead a Cournot oligopoly model where firms are allowed to perform R&D investment to obtain future reductions in marginal costs.

These differences at the level of the formalization strategy are important in two ways. First, because it allows to test NEG previsions against a different market structure, namely oligopoly. Second, because we look at other types of firms strategies than only choosing prices or quantities, namely investment in R&D. In fact, one of the most important characteristics of modern industrial organization is that firms try to influence market behavior through strategic variables as R&D investment. This is especially important in high-tech sectors. Moreover, international competition between firms is more and more centered also in R&D competition (besides output and price competition). If that is so, it is important to analyze in what ways this type of strategic behavior can influence market outcomes, industry dynamics and international location.

We confirm NEG result, of the higher propensity of economic activity to concentrate in few locations. Propensity for agglomeration is derived, as in NEG, as result of 'demand linkage effects': the country that attracts a higher proportion of consumption is a more attractive location for firms since it generates higher demand (and therefore profits). However, in our model this propensity for agglomeration is also derived based on 'R&D linkages effects': firms located in the country with more skilled-labor perform higher levels of R&D investment ('home market effects' on R&D) what makes firms in this country more efficient ('competitiveness' effect). This 'R&D linkage effect', allows firms to produce at lower costs and consumers to have access at products at lower prices. Therefore, firms want to locate where skilled-workers are, to have more demand ('demand linkage effects') and to have a larger pool of skilled-labor to become more efficient ('R&D linkages effects').

On the other side, skilled-workers want to locate where skilled-workers and firms are, to have access to hi-tech goods at more competitive prices ('price effects'), and to work in what they are specialized, i.e.: R&D activities ('job-match effect'). The 'price effects' arise both because the country with more industry have more locally produced goods that do not incur in trade costs, and the country with more skilled-labor allows firms to perform higher levels of cost-reducing R&D investment. The 'job-match effect' arise in the country with more firms because there are a higher demand for skilled-labor, but can also arise in the country with more skilled-labor, because in this country, firms perform higher levels of R&D investment, that can attract more firms and therefore increase the demand for skilled-labor.

This can lead to 'agglomeration' and 'cumulative causation effects': if countries are asymmetric in terms of labor markets (i.e.: if one country has more skilled-labor than the other), then firms become endogenously asymmetric (because firms perform different levels of R&D). This means that more efficient firms are locate in the country with more skilled-labor, what promotes appearance of new firms in this country and the attraction of more skilled-labor. This cycle can be repeated and lead to 'cumulative causation effects' that can feed agglomeration processes. In fact, if one country has conditions to host more efficient firms, this will mean that delocalization of firms is promoted to this location, and that firms in the other country will face fiercer competition from these firms. Since asymmetric equilibriums are unstable, given that these promote migration of skilled-workers, and delocalization of firms to the more competitive country, it can happen that a stable equilibrium is only reached when all skilled-workers and hi-tech firms are located in the same location.

Namely dispersion is only promoted for high levels of trade costs, high levels of competition, high cost-reducing effect of R&D investment; and for low levels of market size, and low levels of costs of R&D investment. High levels of trade costs, and low levels of market size make exports too costly and as such firms prefer to be closer to markets. High levels of competition promotes symmetric equilibriums because dispersion attenuates local competition. High cost-reducing effects of R&D, and low costs of R&D makes competition fiercer (since these turn firms more efficient) what discourages agglomeration.

As such, we showed that for policy makers and authorities responsible for industrial and regional policy, R&D investment must be a central issue in their concerns. For less developed regions, policies that promote the reduction of the cost of R&D investment and the increase in the effectiveness of R&D investment are important to attract industry and skilled-workers, and therefore to avoid centre-periphery patterns and agglomeration.

Our model also shed some light on the problematic of 'brain drain'. We showed that even when wage differentials are inexistent (or negligible), the countries that perform higher levels of R&D investment attract more skilled-labor. Furthermore, this can also be a 'cumulative causation' phenomena: firms located in the country that attracts more skilled-labor perform higher levels of R&D investment, what in turn promotes the attraction of more skilled-labor since by increasing R&D, welfare also increases, and so on. The policy implication of this conclusion is that a country in order to attract and fixate skilled-labor should promote R&D investment by firms.

In terms of the phenomena of delocalization, we showed that this is a spatial dynamic phenomena since it depends on the comparative performance of firms between different locations. This phenomena is related in one turn with R&D investment, and in another turn with spatial skilled-labor dynamics. In fact, local R&D performance and local labor markets can affect the performance of firms in other locations. Then R&D investment can promote delocalization of firms to countries where high levels of R&D is perform, since firms want to be located in the region that allows them to be more efficient. Therefore we have something of the type of strategic location choices, since firms choose locations in order to affect the performance and competitiveness of themselves and competitors.

This happens to be so because spatial labor patterns, can turn firms asymmetric through the R&D channel, since the country with more skilled-labor allows firms to perform higher levels of R&D investment. Remember that if skilled-workers are evenly distributed between locations, firms are symmetric because they perform the same levels of R&D. Only if skilled-workers are asymmetrically located, firms can endogenously differentiate themselves (i.e.: to become asymmetric) at the level of R&D investment (and as consequence at the level of marginal costs, prices and in the end profits). Therefore, in order to attract firms, policy makers from peripheral countries should promote both R&D and training of skilled-labor.

11 References

Bertola, G. (2000), "Labor Markets in the European Union", *Ifo-Studien*, 46, pp. 99-112.

Blanchard, O. and Katz, L. (1992), "Regional Evolutions", *Brooking Papers on Economic Activity*, 1, pp.1-75.

Brander, J. (1981), "Intra Industry Trade in Identical Commodities", *Journal of International Economics*, 11, pp. 1-14.

Dixit, A. and Stiglitz, J. (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review*, 67, pp. 297-308.

Fujita, M.; Krugman, P. and Venables, A. (1999), "*The Spatial Economy: Cities, Regions, and International Trade*", Cambridge: MIT Press.

Head, K.; Mayer, T. and Ries, J. (2000), "On the Pervasiveness of Home Market Effects", Mimeo CERAS-CNRS.

Krugman, P. (1980), "Scale Economies, Product Differentiation, and the Pattern of Trade", *American Economic Review*, 70, pp. 950-959.

Krugman, P. (1991), "Increasing Returns and Economic Geography", *Journal of Political Economy*, 99, pp. 413-499.

Krugman, P. and Venables, A. (1995), "Globalization and the Inequalities of Nations", *Quarterly Journal of Economics*, 110, pp. 857-880.

Leahy, D. and Neary, P. (1996), "International R&D Rivalry and Industry Strategy without Government Commitment", *Review of International Economics*, 4, pp. 322-338.

Neary, P. (2001), "Monopolistic Competition and International Trade Theory", Mimeo UCD.

Ottaviano, G.; Tabuchi, T. and Thisse, J.-F. (2001), "Agglomeration and Trade Revisited", forthcoming *International Economic Review*.

Pires, A. J. G. (2003), "R&D Investment, International Trade and Home Market and Competitiveness Effects", mimeo, ISEG/UTL.

Puga, D. (1999), "The Rise and Fall of Regional Inequalities", *European Economic Review*, 43, pp. 303-334.

Samuelson, P. (1954), "The Transfer Problem and Transport Costs", *The Economic Journal*, 64, pp. 264-289.

Spencer, B. and Brander, J. (1983), "International R&D Rivalry and Industry Strategy", *Review of Economic Studies*, 50, pp. 707-722.

Venables, A. (1996), "Equilibrium Locations of Vertically Linked Industries" *International Economic Review*, 37, pp. 341-359.

12 Appendix

In this section we give mathematical proofs for some of the relations presented in the text.

Sign of ϕ Remember again the expression for R&D investment by home firms (equation 16):

$$k = \frac{\theta M}{\phi}\left(\left(\theta^2 M - \gamma\right)\left(D - t\left(1-r\right)\right) - 2\gamma N t\left(1-s\right)\left(r - \frac{1}{2}\right)\right)$$

With $\phi = \varphi\left(\theta^2 M - \gamma\right)$, and where $\varphi = \left(\gamma\left(N+1\right) - \theta^2 M\right)$. It can be easily seen that ϕ is quadratic in γ. The two associated solutions are $\gamma = \theta^2 M$ and $\gamma\left(N+1\right) = \theta^2 M$. Therefore, for having a solution for k, we can not have $\gamma = \theta^2 M$, or $\gamma\left(N+1\right) = \theta^2 M$, since then the denominator is zero and the whole expression has no solution. Note also, that ϕ is concave in γ ($\frac{d^2\phi}{\gamma^2} = -2\left(N+1\right)$). As such, for $\gamma < \frac{\theta^2 M}{N+1}$, and $\gamma > \theta^2 M$, $\phi < 0$; on the contrary for $\frac{\theta^2 M}{N+1} < \gamma < \theta^2 M$, $\phi > 0$. The question is what is the case that applies in our model.

We can investigate this by looking at the determinant implied by the R&D investment system. We want this determinant to be positive. The determinant can be found by using the general conditions for k and k^* (equation 15) and substitute for q, q^*, x and x^* (equation 14). In matrix form we will have:

$$\begin{bmatrix} \gamma - \frac{\theta^2 M}{N+1}\left(\left(1-s\right)N+1\right) & \frac{\theta^2 M}{N+1}\left(1-s\right)N \\ \frac{\theta^2 M}{N+1}sN & \gamma - \frac{\theta^2 M}{N+1}\left(sN+1\right) \end{bmatrix}\begin{bmatrix} k \\ k^* \end{bmatrix} = \begin{bmatrix} \frac{\theta M}{N+1}\left(D + t\left(r\left(1-s\right)N - \left(1-r\right)\left(\left(1-s\right)N+1\right)\right)\right) \\ \frac{\theta M}{N+1}\left(D - t\left(r\left(sN+1\right) - \left(1-r\right)sN\right)\right) \end{bmatrix}$$

The determinant of this system is just $\left(\gamma\left(N+1\right) - \theta^2 M\right)\left(\gamma - \theta^2 M\right)$ or $-\phi$. Then, it comes out that the determinant is positive for $\gamma < \frac{\theta^2 M}{N+1}$ and for $\gamma > \theta^2 M$. Therefore, we can rule out the solution $\frac{\theta^2 M}{N+1} < \gamma < \theta^2 M$.

Note however, that for $\gamma < \frac{\theta^2 M}{N+1}$ the model can predict negative levels of R&D investment. To see this, note that the multiplicative term in equation (16) is always negative. On the contrary the first term on parenthesis is always positive as long as the OMC for the SCM is satisfied. Then it comes, that we have negative levels of R&D investment for $r \leq \frac{1}{2}$, since the second term in parenthesis is positive for $r < \frac{1}{2}$, and zero for $r = \frac{1}{2}$ (making the all expression in parenthesis positive). Therefore, we also want to rule out the case where $\gamma < \frac{\theta^2 M}{N+1}$, since we are not interested in negative levels of R&D investment (at most firms do not invest).

On the contrary, if $\gamma > \theta^2 M$, the model can predict positive levels of R&D investment. In fact, we have again that the multiplicative term is negative, but now the first term on parenthesis is also negative. Therefore, for $r \geq \frac{1}{2}$ the model predicts that all firms have positive levels of R&D investment, since the second term in parenthesis is zero for $r = \frac{1}{2}$ and negative for $r > \frac{1}{2}$. For $r < \frac{1}{2}$ we have that the second term in parenthesis is positive. Therefore, to have positive levels of R&D investment we will need that the first term in parenthesis is bigger than the second one, or that the relation between market size and the level of trade costs and R&D investment variables is such that:

$$D > \frac{t}{\theta^2 M - \gamma}\left(\theta^2 M(1-r) + \gamma\left(2N(1-s)\left(r - \frac{1}{2}\right) - (1-r)\right)\right)$$

We can easily see that if $\gamma > \theta^2 M$ this condition is always negative for $r > \frac{1}{2}$. To see this note that the multiplicative term is negative and that the condition is linear in γ. Then note also, that at $\gamma = \theta^2 M$, the condition simplifies to $2\theta^2 MN(1-s)\left(r - \frac{1}{2}\right)$. Therefore, for $r > \frac{1}{2}$ this condition is always satisfied since $D > 0$. On the contrary, for $r < \frac{1}{2}$ the condition is positive and as such for this condition to be satisfied, D needs to be superior to the value implied by the condition above.

Then, for having $k > 0$, we need that $\gamma > \theta^2 M$ (what implies $\varphi > 0$), and that market size is sufficiently big to satisfy the R&D condition.

Sign of the Indirect Utility Function (RDM) In the RDM the indirect utility differential is:

$$\Delta V = \frac{2N^2 t}{\varphi (N+1)^2}\left(s - \frac{1}{2}\right)\left(\gamma (N+1)\left(D - \frac{t}{2}\right) + 2\theta^2 Mt\left(s - \frac{1}{2}\right)\left(r - \frac{1}{2}\right)\right)$$

To see that, if $s > \frac{1}{2}$, $\Delta V > 0$ (and the contrary for $s < \frac{1}{2}$), first note that if both $\left(s - \frac{1}{2}\right)$ and $\left(r - \frac{1}{2}\right)$ are of the same sign (i.e.: $\left(s > \frac{1}{2}\right)$ and $\left(r > \frac{1}{2}\right)$; $\left(s < \frac{1}{2}\right)$ and $\left(r < \frac{1}{2}\right)$; or $\left(r = \frac{1}{2}\right)$), then the indirect utility differential unambiguously increases with the home share of firms (since the multiplicative term is always positive and what is inside parenthesis also becomes always positive as long as the OMC for the SCM is satisfied).

For $\left(s > \frac{1}{2}\right)$ and $\left(r < \frac{1}{2}\right)$; and $\left(s < \frac{1}{2}\right)\left(r > \frac{1}{2}\right)$ we need further qualification. Then note that at $\gamma = \theta^2 M$, the expression in parenthesis simplifies to $\theta^2 M\left(D(N+1) - t\left(\frac{1}{2}(N-1) - 4s\left(r - \frac{1}{2}\right) + 2r\right)\right)$, and this is always positive as long as the OMC or the SCM is satisfied, and $N > 2$ (as is the case in our model). Therefore, and since the expression in parenthesis is linear in γ, what is inside parenthesis is also always positive for $\gamma > \theta^2 M$.

Spatial Behavior of the Indirect Utility Differential (RDM) Derivative of the indirect utility function in relation to local share of firms:

$$\frac{d\Delta V}{ds} = \frac{2N^2 t}{\varphi (N+1)^2}\left(\gamma (N+1)\left(D - \frac{t}{2}\right) + 4\theta^2 Mt\left(s - \frac{1}{2}\right)\left(r - \frac{1}{2}\right)\right)$$

Before signing the derivative of indirect utility differential in relation to s, we make some remarks. First, if both $\left(s - \frac{1}{2}\right)$ and $\left(r - \frac{1}{2}\right)$ are of the same sign (i.e.: $\left(s > \frac{1}{2}\right)$ and $\left(r > \frac{1}{2}\right)$; $\left(s < \frac{1}{2}\right)$ and $\left(r < \frac{1}{2}\right)$; or $\left(s = \frac{1}{2}\right)$ or $\left(r = \frac{1}{2}\right)$), then this derivative unambiguously increases with the home share of firms (since the multiplicative term is always positive and what is inside parenthesis also becomes always positive as long as the OMC for the SCM is satisfied, and $\gamma > \theta^2 M$).

For $\left(s > \frac{1}{2}\right)$ and $\left(r < \frac{1}{2}\right)$; and $\left(s < \frac{1}{2}\right)\left(r > \frac{1}{2}\right)$ we need further qualification. Then note that at $\gamma = \theta^2 M$, the expression in parenthesis becomes $\theta^2 M \left(D\left(N+1\right) - t\left(\frac{1}{2}\left(N-1\right) - 4s\left(r - \frac{1}{2}\right) + 2r\right)\right)$, and this is always positive as long as the OMC for the SCM is satisfied. Therefore, and since the expression in parenthesis is linear in γ, what is inside parenthesis is also always positive for $\gamma > \theta^2 M$. As such this derivative increases with s.