Linear Spatial Analysis of Complex Socio-Economic Systems

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Abstract. In this paper we consider the unifying framework for partially ignored and neglected and partially misunderstood and misinterpreted linear spatial socioeconomic theories. The paper includes
the following three themes connected with the analysis of spatial socio-economic theories:

- the sensitivity analysis and catastrophes of the optimal solutions of the linear programming and the transportation problem,
- the theory of migration streams,
- the spatial production cycles and trade feedback loops.

1. **Introduction**

In this paper, we will concentrate ourselves on the forms of complication and self-organization in linear spatial socio-economic systems. The description of the catastrophic effects of appearance of new information and new emerging properties in the states of the linear spatial socio-economic systems will be in a center of our considerations.

The important predecessor in studies of linear economic structures was Goodwin (1983), who stated that

> *"...economies are so impossibly complex as to defy any completely satisfactory analysis: rather the best can be hoped for a number of different approaches, each of which yields valuable but incomplete insights into the various aspects of the system. Thus 'general equilibrium' theory, whilst in principle admirable, always has seemed to me to be so 'general' as to be largely vacuous and even capable of diverting attention from important matters. By contrast Keynesian theory and cycle theory, for all their crudity, attracted me because of the usable practical results. Yet they lump together inhomogeneous elements into aggregates, which cannot be expected to yield empirically valid relationships. Making some sort of practical compromise by disaggregating, one is bound to be dealing with a complicated system with a large number of variables. A large system can yield little results unless it is linear. Therefore I was attracted to linear systems and in particular to the simplicities of Leontief's input-output method which appeared to be capable of demonstrating some of the virtues as well as some of the faults of both the Keynesian and the Walrasian types of analysis…. The neoclassical economics made a tangible advance in the subtlety and realism of their analysis, but they tended to lose sight of the larger issues and sweep of the classical economists. Linear disaggregated systems seem to me to constitute a*

> *fruitful compromise between the virtues of these two divergent methodologies"* (Goodwin, 1983, Preface, p. vi.).

The second important source of the analysis of linear economic structure may be attributed to the famous book of David Gale "The Theory of linear economic models" (Gale (1960). In this book, he concentrated on linear programming, game theory, aspects of general linear algebra that were relevant to linear economic analysis, and linear models of production. In this chapter, we will add some additional insights that build on the frame of Gale's scheme, namely sensitivity analysis of the classical linear programming problem in polyhedral form; the economic interpretation of the theory of convex polyhedrons in the form of the superposition principle of analysis of extreme tendencies, emerging in the evolving linear economic systems.

The new analysis can be considered as a recognition and reflection of a new phenomenon in the field of emergence and organization of scientific paradigms: the theorists of the mainstream economics once more found the existence of economic and urban geography and spatial analysis. Forty-five years after Isard's (1956) seminal book M Fujita et al. (1999) provided some new perspectives. In the latter book, the reasons for the neglect of spatial theories by mainstream economics were determined: the absence of unifying framework of theoretical models as well as the absence of standard mathematical modeling tricks. It should be noted that the negligence was two-sided: other spatial analysts formulated and resolved separate theoretical and practical problems, connected not with control and optimization, but with the analysis of actual states.

Further, this paper will point out the deficiency of purely economic considerations of socio-economic systems and will stress the necessity to widen the concept of "Homo Oeconomicus" to the concept of "Homo Socialis." Such a widening is radical in the study of complex socio-economic processes because of the important difference between the economic and socio-economic rationality: the traditional identification of economic rationality of "Homo Oeconomicus" as optimization (Arrow, 1963, p.3) is complimentary to socio-economic rationality of "Homo Socialis" as parsimony (Sonis, 2001, pp. 330). Hence, this paper stresses the necessity to transfer from economic optimization by considering the superposition of different optimization tendencies and analysis of concrete (or realizable) states of socio-economic systems.

2. Catastrophe effects in Linear Programming

We will start our consideration of linear socio-economic systems from the classical linear programming optimization problem (see Dantzig, 1963; Dorfman, Samuelson and Solow, 1958; Gass, 1961). Since linear programming problem has became very well known, we will concentrate only on part of this theory connected with the process of complication based on the behavior of optimal solutions and their cone-wedge domains of structural stability and the "catastrophe" effects of structural changes.

2.1. Cone-Wedge presentation of the domain of Structural Stability of optimal solutions.

The domain of stability of the basis of optimal solutions in linear programming is the aggregation of two different domains: (1) the domain of permissible changes of the resources (free coefficients of the system of linear constraints); and (2) the domain of admissible changes in prices (coefficients of the objective function), under which the optimal solutions of the linear programming problem will correspond to the same basis, i.e., to the same set of non-zero components of optimal solutions.

In essence, the conditions of structural stability are the conditions of the preservation of the optimal assortments of production in the linear economic system or conditions of optimal organization of space in a spatial system. The potential link with input-output analysis provides for the intriguing possibilities of exploring ways in which prices (or quantities) can be used as a tool for the optimal management of an economic system undergoing technological changes or for a system of regions facing changing competitive pressures. In other words, this section focuses on the description of the sensitivity analysis of the optimal solutions of the linear programming problem under conditions of unchanging technology. This implies that the coefficients of the objective function and the right parts of the system of linear inequalities are arbitrarily changing. We chose such form of sensitivity analysis, which describe the catastrophe changes (sudden jumps) in optimal solutions structure.

The description of these effects is based on the polyhedral form of general sensitivity analysis for the classical linear programming problem (see Sonis, 1982a):

Consider a primal linear programming problem **LP** *and an associated dual problem* **D**:

$$\text{LP:}\quad \begin{cases} AX = b \\ X \geq 0 \end{cases} \qquad \text{D:}\quad YA \leq c \qquad (2.1)$$
$$cx \rightarrow \min \qquad\qquad Yb \rightarrow \max$$

Let A_0 *be an invertible sub matrix of the matrix A with the inverse* $A_0^{-1} = B$ *with the properties:*

$$B \geq 0, \qquad c_0 BA \leq c \qquad (2.2)$$

where the coordinates of the vector c_0 *correspond to the columns of the matrix* A_0. *Then the primal problem has the optimal solution, X, with the vector of non-zero basis components* X_0

$$X_0 = Bb \qquad (2.3)$$

and the dual has the optimal solution:

$$Y = c_0 B \qquad (2.4)$$

This proposition also provides the complete description of the domains of the structural stability of the optimal solutions for the primal and dual linear programming problems under conditions of unchanging technology: if the resources, *b*, and prices, *c*, are changed, a polyhedral *cone* in the spaces of resources:

$$C = \left(b : Bb \geq 0\right) \tag{2.5}$$

and a polyhedral *wedge* in the space of costs:

$$W = \left(c : c_0 BA \leq c\right) \tag{2.6}$$

are obtained. Thus, the Cartesian product $C \times W$ defines the domains of the structural stability of the optimal solutions for the primal and dual problem. The construction of the Cartesian product for each given optimal solution is simple, because the last tableau of the simplex algorithm of Dantzig, 1963, contains the components of the matrix BA. Hence, to obtain the inequalities determining the domain of the structural stability, access to the components of the last simplex tableau will suffice. Moreover, the optimal solutions, X and Y, associated with the basis matrix A_0 are vertices of the corresponding convex polyhedrons of the admissible solutions for the primal and dual problems. Since the matrix A contains a finite number of invertible sub matrices, the space of resources and the space of costs are decomposable into a finite number of domains:

$$C_1 \times W_1, C_2 \times W_2, ..., C_r \times W_r \tag{2.7}$$

so that each of them corresponds to the preservation of some invertible basis sub matrix of the matrix A, i.e., to the preservation of some optimal assortment of production.

The transition from the domain $C_i \times W_i$ to the next domain $C_{i+1} \times W_{i+1}$ may be described as the crossing of one of the bounds of the cone C_i or wedge W_i. In this case, outside the cone, C_i, the criterion of optimality will fail to hold in the cell of the objective row of the simplex tableau corresponding to the bound of the transition. This cell defines the type of production to be introduced into the basis and for construction of the next cone C_{i+1} only one step of the simplex algorithm is needed.

If the transition through the bound of the wedge, W_i, takes place, then the condition of positivity of the components of the optimal solution fails to hold in the row corresponding to the chosen bound. This bound defines the type of production to be eliminated from the basis of the solution and, as before, only one further step is necessary in the dual simplex algorithm.

3. Structure of optimal (minimal cost) transportation flows

This section presents the application of the results of the sensitivity analysis of the general linear programming problem to the classical transportation minimal cost problem (Danzig, 1951). The actual hierarchy of urban settlements puts strong restrictions on the spatial organization of optimal (minimal cost) transportation flows between the settlements. In turn, the spatial and temporal stability of the transportation flows may be the essential factor of growth or decline of a hierarchy of urban settlements.

3.1. Domains of structural stability and boundaries of structural change in optimal transportation networks.

Consider the cost minimization problem on a network with *m* suppliers with a_i units of supply for each of the suppliers (*i*= 1,...,*m*) and *n* demanders with corresponding needs for b_j units (*j*=1,...,*n*) such that the total supply is equal to total demand: $\sum a_i = \sum b_j$, and let c_{ij} be the cost of transportation of one unit of production from the i^{th} supplier to the j^{th} demander. The description of the domains of the structural stability provides the mechanism for finding the optimal linkages between demanders and suppliers (see Sonis, 1982a). The difficulty here is that the solution to the transportation problem does not provide the last simplex tableau and it must be restored. For the re-establishment of the matrices, *BA* and *B*, a generalization of the MODI method is used (Dantzig, 1951), providing a connection with the simple structure of the matrix associated with the transportation problem. To simplify the presentation, a description of the procedure for the construction of the domain of the basis stability will be provided (see Sonis, 1982a, 2000a).

3.1.1. Vector Method of Potentials and matrix inequalities of Cone-Wedge domains of structural stability of optimal networks.

Construct the computation table, which includes *m* rows, corresponding to the fixed supply $a_1, a_2, \ldots, a_m$, and *n* columns, corresponding to the fixed consumption (demand) $b_1, b_2, \ldots, b_n$, and *mn* transportation costs c_{ij}. First, the transportation problem should be solved; the occupied cells (basis cells) of the computation table provide us with the components of the cost vector, c_0. The matrix BA is constructed in the following way: in the basis cells, insert the columns (unit vectors):

$$e_k = (0,0,\ldots,1,\ldots 0)^T \quad , \qquad k = 1,2,\ldots,n+m-1 \tag{3.1}$$

of the identity matrix of order *m+n-1* one after the other.

Next, a following set of vector-potentials is identified:

$$U_1 = (0,0,\ldots 0)^T, \qquad U_2,\ldots,U_m;\ V_1,V_2,\ldots,V_n \tag{3.2}$$

such that $U_i + V_j = e_k$ where e_k is the unit vector corresponding to the basis cell (*i,j*). In the nonbasis cells, (i_0, j_0), of the computation table, insert the vectors $U_{i_0} + V_{j_0}$. Once this is accomplished, the content of the computation table will be composed of the columns of the matrix BA. Since the objective function in the transportation program is to be minimized, results from section 2 can be used for the construction of the wedge, *W*, using the inequality $c_0 BA \le c$ according to which the matrix B can be restored.

Recall that the initial simplex matrix of the transportation problem is:

$$\begin{array}{c|ccccccccccccc}
 & x_{11}, & x_{12}, & \ldots & x_{1n}, & x_{21}, & x_{22}, & \ldots & x_{2n}, & \ldots & x_{m1}, & x_{m2}, & \ldots & x_{mn} \\
\hline
1 & 1 & 1 & \ldots & 1 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\
\vdots & 0 & 0 & \ldots & 0 & 1 & 1 & \ldots & 1 & \ldots & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
m & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & \ldots & 1 & 1 & \ldots & 1 \\
m+1 & 1 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & \ldots & 1 & 0 & \ldots & 0 \\
m+2 & 0 & 1 & \ldots & 0 & 0 & 1 & \ldots & 0 & \ldots & 0 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
m+n & 0 & 0 & \ldots & 1 & 0 & 0 & \ldots & 1 & \ldots & 0 & 0 & \ldots & 1
\end{array} \qquad (3.3)$$

If we eliminate the last linear dependent row of this matrix, then it is easy to see that in the columns corresponding to $x_{1n}, x_{2n}, \ldots, x_{mn}$ the first *m* columns of the identity matrix are situated. To obtain the columns, the n^{th} column must be subtracted from the *n-1* columns; the columns, labeled *1n, 2n,..., mn* correspond with the cells (*1,n*),..., (*m,n*) of the computation table, then the first *m* columns of the matrix *B* are the sums $U_1 + V_n,\ U_2 + V_n, \ldots, U_m + V_n$ and the following *n-1* columns are the differences $V_1 - (U_1 + V_n), V_2 - (U_1 + V_n), \ldots, V_{n-1} - (U_1 + V_n)$. Since $U_1 = (0)$, the following may be obtained:

$$B = \left(V_n, V_2 + V_n, \ldots, U_n + V_n, V_1 - V_n, V_2 - V_n, \ldots, V_{n-1} - V_n\right) \qquad (3.4)$$

In this case, the cone, *C*, is $Bb \geq \bar{0}$ where: $b = \left(a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_{n-1}\right)^T$

(3.5)

and the optimal solution in the domain $C \times W$ is $X_0 = Bb$.

3.1.2. Structural change in the spatial structure of optimal transportation flows.

The changes in the spatial structure of the optimal transportation flow are connected to the absence of fulfillment of one or more of the inequalities defining the cone and wedge of the structural stability. The domains of the structural change are the faces of the domain of the structural stability $C \times W$, which are the closed hyperplanes in the supply-demand space or in the space of transportation costs. On the face of the cone *C*, the flow is degenerated; it divides into a few independent sub flows that are the optimal solutions for a smaller size problem. If one moves out of the cone *C*, then the admissible flow with a given topological structure does not exist and a new flow must be constructed. If one moves out of the wedge *W*, then there is an admissible flow with a previous topological structure, but the condition of optimality of the transportation flow fails to hold, and the structure of the flow must be changed by substituting one arc of the spatial structure for another.

3.2 Behavioral competition between suppliers and demanders within the minimal cost

transportation problem.

In this subsection, it is shown that in the minimum cost transportation problem the *global collective minimization of cost*s implies the totally antagonistic *competitive exclusion individual behavior* of suppliers and demanders.

3.2.1. Competitive exclusion behavioral rules in the minimum cost solution.

It is well known in the linear programming transportation problem that the competitive forces that result in an optimal allocation may lead to the exclusion of some interconnection between some subgroups of suppliers and demanders. This effect will now be explored in the form of behavioral rules for subsets of suppliers and demanders. Consider an arbitrary subset of all the basic cells. The suppliers and demanders in this subset will be referred to as the *old* suppliers and demanders and the complement set of suppliers and demanders will be referred to as the *new* suppliers and demanders. The following three rules comprise the competitive exclusion effect (see Sonis, 1993b, 2000a):

1. each *new* demander can be served by only one *old* supplier;
2. each *new* supplier can serve only one *old* demander;
3. if a *new* demander is served by both *old* and *new* suppliers, then this *new* supplier cannot serve any other *old* demander.

These behavioral rules allow for the construction of the geometric and numeric algorithm of enumeration of all basic subgraphs presenting the spatial structure of the transportation network carrying the optimal transportation flows under various requirements stipulated by the supply-demand relationships and transportation costs. This essentially simplifies the basic sub graph enumeration procedure.

These behavioral rules reflect the characteristical feature of any global optimization: the global optimization on the macro-level can be achieved only by tough competition and competitive exclusion on the micro-level. This feature of global optimization schould be taken seriously by planners and decision-makers.

4. Superposition Principle – the inverted problem of Multi-objective Programming.

In this section, we contrast the optimization concept that is a central idea of all modern economic analysis, with the concept of the superposition of different extreme optimal tendencies, developing within a given economic system. In such a way, we challenge the idea of optimization with the idea of analysis of *actual* economic systems. The optimization concept is the quintessence of economic rationality (Arrow, 1990). The bounded rationality concept (Simon, 1957) recognized the cognitive limitations of both knowledge and computational capacity of economic decision-makers. In the actual states of complex system, the different behavior of many actors with bounded rationality generates the existence of different optimization tendencies and their superposition. In the complication process, with a flow of new emerging tendencies, the economic rationality is replaced by the concept of parsimony (Sonis, 2000b). The concept of parsimony needs the evaluation of a set of possible extreme tendencies and a measure of their realization in actual states of evolving complex system. So the concept of parsimony replaces the concept of optimization by the superposition analysis of existing extreme tendencies (Sonis, 1982b). The concept of parsimonity is the essential feature of the choice behaviour of "social man" ("Homo Socialis"}, the notion coined by Perroux (1964)**, (**see also Sonis, 1992). "Homo Socialis" is the " collective being", which cannot exist and survive without and outside of society (collective). He has no full information about all possible choice alternatives, he do not know about the utility properties of these choices and has no knowledge about the form of his utility function. The information about the choice alternatives and their utility he obtains through the learning process. This learning process includes:

- Imitation of choice behavior of "near peers".
- Extraction of information about possible choices and their utility through direct contacts (social interaction) within an active, uncertain environment with the "near-peers" and through mass media presenting "ready" opinions and solutions and making difficult the rational evaluation of choices and their utilities: each person, who did a choice became "a specialist", rational, wise or not, heavily influencing the subjective mental evaluation of marginal spatio-temporal utilities (expectations of gains in the future or in other location).
- The gathering the information about possible choices needs formidable efforts, such that the adopters of new choice alternatives can be divide into two major behavioral groups: satisfiers, who are stopping their search when the rate of marginal growth in the gain in the utility is not justified by the effots, and maximizers, who are checking all possible choice alternatives, known to them (see recent book by Schwartz, 2004)
- "Learning by using" the chosen innovation.

The subjective mental expectations of gains in the future or in other location represent the main propensity of Homo Socialis towards the parsimony in efforts and expenses. This thrift propensity replaces for Homo Socialis the utility maximization principle(see Sonis, 1981, 1991, 1992, 2000c, 2001).

4.1. Connection between the Weber principle of industrial location and the Möbius Barycentric Calculus.

Geometrically, the solution of the linear programming optimization problem takes into account only one vertex of the convex polyhedron of all admissible solutions. The information about the set of all other vertices and the structure of the convex polyhedron, while it is important for the derivation of the solution, is neglected in the optimal solution itself. Moreover, the actual state of the linear regional system (a system defined by linear balancing constraints) is usually far from the optimization. From the viewpoint of optimization, the actual state of a regional system is a solution for a multi-objective programming problem. This means that the actual state reflects the existence of a set of different extreme tendencies or trends corresponding to the optimization of a set of different objective functions. Simultaneous optimization of two or more objective functions is inaccessible mathematically (Boltiansky, 1973, paragraph 1.5). Therefore, the problem of multi-objective programming is usually transformed to the problem with only one objective. Traditionally there are two approaches for this transformation (Cohon, 1978). One of them is to optimize one of objectives while appending the other objectives to a constraint set, so that the (sub-optimal) solution would satisfy these objectives up to an acceptable level This is what is done usually in thr entropy maximization approaches – with either minimization of transportation costs as objective and entropy as a constraint oe the other way around (see Wilson (1970), Erlander(1980), Anas (1983, Erlander and Stewart (1990)). The other approach is to optimize a super-objective function created by weighted sum of a set of objectives (see Casetti, 1972, Nijkamp, 1986, Roy, 1990, Wegener, 1994, among others). There is a great deal of arbitrariness in both approaches and the influence of each objective is distorted; therefore, the optimal solution of the multi-objective programming is usually far removed from the actual state of the regional system.

The problem becomes much easier if we replace the consideration of multi-objective optimization with the problem of analysis of an actual state of linear regional system. Geometrically, the actual state belongs to the convex polyhedron of admissible solutions; the vertices of this polyhedron are the optimal solutions of one objective optimization problems. So we find ourselves in the typical situation of the theory of convex polyhedrons: a point (of actual state) within the convex polyhedron (of admissible solutions). The central fact of the theory of convex polyhedrons is the Minkovski, (1910), theorem about the center of gravity of a convex polyhedron: it is possible to hang the collection of weights (with common weight 1) on the vertices of the convex polyhedron such that its center of gravity will coincide with a given point. More precisely, the Minkovski theorem can be can be formulized in the following manner: *every point Y_1 of a convex bounded many-dimensional polyhedron can be presented as a convex combination (a weighted sum) of several vertices $X_1, X_2, ..., X_k$:*

$$Y_1 = p_1 X_1 + p_2 X_2 + ... + p_k X_k, \quad 0 \le p_i \le 1, \ i = 1, 2, ..., k, \ p_1 + p_2 + ... + p_k = 1 \quad (4.1)$$

The Minkovski theorem can be interpreted as an inversion of the classical Weber's principle of industrial location (Weber, 1909). Weber's main idea was the utilization of the notion of center of gravity: the optimal location of a plant is the center of gravity of a polygon whose vertices correspond to the location of raw materials, energy, manpower and to the market location. We shall use the following inversion of Weber's principle*: the point of the actual state of the regional system is considered as a center of gravity of the polyhedron of admissible states of the regional system.* So we determine the collection of vertices X_i and their weights (barycentric coordinates) p_i such that the center of gravity of the polyhedron of admissible states will coincide with the actual state. Thus, the problem of analysis of an actual state of the regional system is reduced to the basic problem of Barycentric Calculus (Moebius, 1827).

4.2. The Caratheodory theorem and the inverted problem of multi-objective programming.

It is important to note that in the decomposition (4.1) it is possible to use only a subset of the vertices of the convex polyhedron belonging to some simplex (a multi-dimensional pyramid). This is the content of the specification of the Minkovski theorem established by Caratheodory, (1911): *every point Y_1 within a convex closed bounded n-dimensional polyhedron can be presented by a convex combination of vertices, $X_1, X_2, ..., X_{m+1}$, belonging to some m-dimensional simplex ($m \le n$) with m+1 vertices*:

$$Y_1 = p_1 X_1 + p_2 X_2 + ... + p_k X_{m+1}, \quad 0 \le p_i \le 1, \; i = 1, 2, ..., m+1, \; p_1 + p_2 + ... + p_{m+1} = 1 \quad (4.2)$$

In other words, the given point Y_1 is a center of gravity of the set of weights $p_1, p_2, ..., p_{m+1}$ hanging on the vertices of certain simplex. Moreover, the barycentric coordinates $p_1, p_2, ..., p_{m+1}$ of Y_1 with respect to a fixed simplex are defined uniquely.

This theorem plays only an auxiliary role in linear optimization theory. In our study, it will be the basis of the superposition principle of our linear analysis; each actual state of the linear system is the superposition of a set of extreme states of the regional system, which are the optimal solutions of the sequence of optimization problems, presenting the simultaneous action of different extreme tendencies within a linear complex system. The weights (barycentric coordinates) of the extreme states define the measure of their realization in the actual state.

In the case of a linear model given by the system of linear constraints the superposition principle can be presented as the inverted problem of multi-objective programming (Sonis, 1982b):

Let Y_1 be an admissible solution of the system of linear constraints:

$$\begin{cases} AX = b \\ X \ge 0 \end{cases} \qquad \textbf{(4.3)}$$

and let

$$f_1(X), f_2(X), ..., f_s(X) \qquad (4.4)$$

be the ordered set of linear or concave objective functions. Then there is the decomposition of Y_1 into convex combination

$$\begin{aligned}&Y_1 = p_1X_1 + p_2X_2 + \ldots + p_sX_s + p_{s+1}Y_{s+1},\\ &0 \le p_i \le 1,\ i = 1,2,\ldots,s+1,\ p_1 + p_2 + \ldots + p_{s+1} = 1\end{aligned} \qquad (4.5)$$

where Y_{s+1} *is the unexplored remainder state and each vector* X_i *is the optimal solution to the optimization problem:*

$$\max\ f_i(x)$$

$$\text{subject to constraints:}\begin{cases}AX = b\\ X \ge 0\end{cases} \qquad (4.6)$$

with additional constraints on coordinates of vector X:

$$x_{k_1} = x_{k_2} = \ldots = x_{k_i - 1} = 0$$

The additional zero constraints correspond to the regional “bottlenecks,” i.e., the parts of the regional system where the competition and conflict between different extreme tendencies obtains its most noticeable form. The ordered set of objective functions (4.4), corresponding to the sequence of extreme tendencies, defines the simplex including the actual state Y_1. Thus, the decomposition (4.5) takes into the consideration of the shares of certain extreme tendencies. So obtaining the decomposition we analyze the actual state from the certain preset viewpoint of the investigator-analyst.

4.3. Decomposition formalism for multi-objective analysis based on Minkovsky-Caratheodory theorem.

Let us consider an actual state $Y_1 = (y_1, y_2, \ldots, y_n)$ of a regional system, which is the positive admissible solution of the system of linear constraints $AX = b$ determining the linear model of a given system. The viewpoint of the investigator-analyst will be given with the help of an ordered set of linear or convex objective functions $f_1(X), f_2(X), \ldots, f_s(X)$ that reflects the extreme tendencies from the investigator viewpoint. The decomposition procedure will be presented in algorithmic form (Sonis, 1982b). The geometrical presentation of the decomposition algorithm can be found in figure 1.

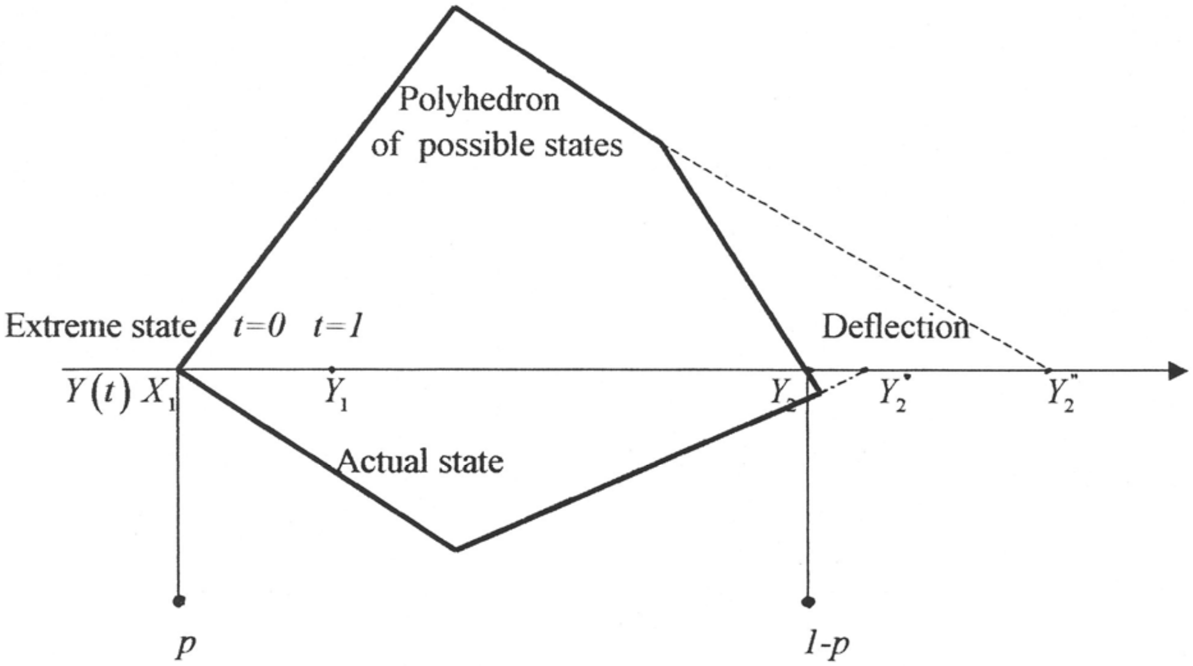

Figure 1. Geometrical illustration of the decomposition algorithm

Step 1. Find the extreme state $X_1 = \left(x_1, x_2, ..., x_n\right)$ that is the full expression of the first extreme tendency.

This first extreme state is the solution to the optimization problem:

$$\max f_1(X)$$

$$\text{subject to constraints:} \begin{cases} AX = b \\ X \geq 0 \end{cases} \qquad (4.7)$$

Two questions arise: what weight p_1 will the chosen extreme state X_1 have in the actual state and in what part of a regional system lies the strongest counteraction to this extreme tendency, which implies the deflection of an actual state from the extreme optimal state?

Step 2. Construct the straight line $Y\left(t\right) = X_1 + t(Y_1 - X_1)$.

Since $Y\left(0\right) = X_1; Y\left(1\right) = Y_1$, this straight line passes through the extreme state X_1 and actual state Y_1 and crosses the opposite face of the polyhedron of admissible states $AX = b, X \geq 0$ at the point Y_2 that represents an unexplored remainder – a deflection of the actual state from extreme state. It is known that if some point lies on the (non-zero dimension) face of a bounded polyhedron defined by the set of non-negative solutions of the system of linear constrains then one of its coordinates is equal to zero. This zero coordinate defines in what part of a regional system there is a “bottleneck” interdiction problem.

Step 3. Find the coordinates of an unexplored remainder.

Find a value of the parameter t (t ≥ 1) that gives some zero coordinate in an unexplored remainder Y_2. Such t is not unique, since the straight line $Y(t) = X_1 + t(Y_1 - X_1)$ can cross the prolongations of several faces of a polyhedron. The required face corresponds to the minimal value

$$t = t_{\min} = \min\{t_k : t_k \geq 1\}$$

where t_k satisfies the equalities $x_k + t_k(y_k - x_k) = 0, \ \ k = 1, 2, ..., n$. Therefore,

$$t_{\min} = \min\left\{\frac{x_k}{x_k - y_k} : x_k \geq y_k\right\} \qquad (4.8)$$

and then

$$Y_2 = X_1 + t_{\min}(Y_1 - X_1)$$

From this equation we find

$$Y_1 = (1 - 1/t_{\min}) X_1 + (1/t_{\min}) Y_2 = p_1 X_1 + (1 - p_1) Y_2 \qquad (4.9)$$

where

$$p_1 = 1 - (1/t_{\min}) = \min\left\{\frac{y_k}{x_k} : x_k \geq y_k\right\} \qquad (4.10)$$

and

$$Y_2 = [1/(1 - p_1)] Y_1 - [p_1/(1 - p_1)] X_1 \qquad (4.11)$$

Thus, we construct the unexplored remainder and the decomposition of the actual state on the share p_1 of the extreme state X_1 and the share $p_2 = 1 - p_1$ of the remainder Y_2. The minimum in (4.10) defines the place of the first "bottleneck" problem in which other extreme tendencies are contradicting (acting against) the first extreme tendency X_1.

Step 4. Find the next extreme tendency and its presentation as the extreme state

The point Y_2 lies on the face of a convex polyhedron. This face is the sub-polyhedron of the lesser dimension. So we find ourselves in the previous situation: we have the convex sub-polyhedron and the point Y_2 within it; we must find in this sub-polyhedron some vertex X_2 corresponding to the optimal solution of the next objective function $f_2(X)$ on the chosen face. On this face one of coordinates (x_{i_1}) corresponding to the place of the first "bottle neck" problem is equal to zero. Thus, the vertex X_2 is the optimal solution of the following problem:

$$\max f_2(X)$$

$$\text{subject to constraints:}\begin{cases} AX = b \\ X \ge 0 \\ x_{i_1} = 0 \end{cases} \tag{4.12}$$

With the help of straight line, crossing the points X_2 and Y_2 we obtain the decomposition

$$Y_2 = q_1 X_2 + \left(1 - q_1\right) Y_3 \tag{4.13}$$

Substituting this into (4.9) we incorporate two extreme tendencies into the decomposition of the actual state:

$$\begin{aligned} Y_1 &= p_1 X_1 + \left(1 - p_1\right) Y_2 = p_1 X_1 + \left(1 - p_1\right)\left[q_1 X_2 + \left(1 - q_1\right) Y_3\right] = \\ &= p_1 X_1 + \left(1 - p_1\right) q_1 X_2 + \left(1 - p_1\right)\left(1 - q_1\right) Y_3 = p_1 X_1 + p_2 X_2 + \left(1 - p_1 - p_2\right) Y_3 \end{aligned}$$

(4.14)

where $p_2 = \left(1 - p_1\right) q_1$ and the second remainder Y_3 includes two zero coordinates corresponding to two "bottle neck" problems within the second remainder Y_3.

Step 5. Repeat the step 4 for the next extreme tendency and etc.

In the same manner we can include in our decomposition the sequence of extreme states appearing in the actual state with the corresponding weights and corresponding "bottlenecks". Since the dimensions of the faces decrease then after no more than *n* steps we will have the final decomposition:

$$Y_1 = \begin{cases} p_1 X_1 + p_2 X_2 + \ldots + p_s X_s + p_{s+1} Y_{s+1}, & p_1 + p_2 + \ldots + p_{s+1} = 1, \quad s < n \\ p_1 X_1 + p_2 X_2 + \ldots + p_n X_n + p_{n+1} X_{n+1}, & p_1 + p_2 + \ldots + p_{n+1} = 1, \quad s \ge n \end{cases} \tag{4.15}$$

where for $s < n$ the remainder Y_{s+1} is the point on (*n*-*s*)-dimensional face of polyhedron and for $s \ge n$ the decomposition includes only *n*+1 extreme tendencies, corresponding to the vertices of *n*-dimensional simplex.

4.3.1. Special case of one linear objective function.

In the case of one linear objective function

$$f_1(X) = f_2(X) = \ldots = f_s(X) \equiv f\left(X\right) \tag{4.16}$$

a numerical procedure of the decomposition can be simplified if we take into consideration the fact that points from *k*-dimensional face include *n*-*k* zero coordinates. Therefore, the choice of consequent extreme states $X_1, X_2, X_3, \ldots$ can be made with the help of the same objective function $f\left(X\right)$ if we replace in this function the coefficients of variables corresponding to zero coordinates in $X_j,\ j = 2, 3, \ldots$ by a very large number

M and solve the M-problem (the linear programming problem with artificial basis, Dantzig, 1963) with the same system of linear constraints (4.3).

5. Polyhedral Catastrophic Dynamics of the Push-Pull states of migration streams.

The Push-Pull analysis of migration streams provides an interesting example of the application of the concept of superposition to the actual state of the migration stream organized in an origin-destination matrix. The Push-Pull concept appeared in what was probably the first migration study by Ravenstein (1885; see also Lee, 1966).

5.1. Description and geometrical interpretation of the decomposition procedure.

This section deals with an analysis and geographical representation of attraction (Pull) and repulsion (Push) in a real migration stream. We restrict ourselves to a detailed representation of the Pull analysis, since the scheme of Push analysis can be considered analogously. For simplicity, we will consider the migration of the homogeneous population of migrants moving within and between the same set of origins/destinations; the consideration of different sets of origin and destinations and the cases of differentiation of migrants by age, sex, nationality, labor specialization, level of education, etc. can be found in Sonis (1980).

Let us consider a homogeneous migrant population moving during a fixed time interval from origins to destinations. This population is statistically described by the origin-destination matrix:

$$M = \left[m_{ij} \right] \qquad (5.1)$$

where $m_{ij} \ge 0, \quad i, j = 1, 2, \ldots, n$ is the number of migrants moving from origin i to destination j. A final distribution of migrants in regions of destination for Pull analysis:

$$K_j = \sum_{i=1}^{n} m_{ij}, \quad j = 1, 2, \ldots, n \qquad (5.2)$$

These data allows for the incorporation of the actual state of the migratory system, M, into the polyhedron of admissible states. For Pull analysis, the convex polyhedron of admissible states includes the migration matrices $X = \left[x_{ij} \right]$, satisfying the following system of linear constraints:

$$\begin{cases} x_{ij} \ge 0, \quad i, j = 1, 2, \ldots, n \\ \sum_{i=1}^{n} x_{ij} = K_j, \quad j = 1, 2, \ldots, n \end{cases} \qquad (5.3)$$

The polyhedron (5.3) is bounded and lies within a many-dimensional rectangular parallelepiped $x_{ij} \le K_j$. The vertices of this parallelepiped are defined by the rule:

"everything or nothing" – their coordinates equal either zero or K_j. This rule has the following geographical meaning (Nystuen and Dacey, 1961): *the extreme tendency represents the repulsion or attraction of migrants only to the region to which the largest number of actual migrants are pushed or attracted.*

The superposition approach decomposes the migration origin-destination matrix *M* into the weighted sum of basis matrices, M_k, representing the action of the extreme tendencies:

$$M = p_1 M_1 + p_2 M_2 + \ldots + p_m M_m \qquad (5.4)$$

where $1 \geq p_s \geq 0$ and $p_1 + p_2 + \ldots + p_m = 1$.

We interpret this decomposition as a display of the principle of intervening opportunities and competition (Stouffer, 1960): *the migrant sees the set of opportunities and selects an opportunity in an attempt to optimize his own objective.* The exchange of the information between the prospective migrants about different opportunities resulted in spatial migration exhibiting empirical regularities (Lee, 1966): "*a migration tends to take place largely within well defined streams*" representing different extreme tendencies. The complete expressions of these extreme tendencies define the assemblage of basis matrices M_s. Each extreme flow M_s enters the real flow *M* with the weight $p_s \leq 1$, and the sum of weights is equal to 1.

The procedure of the Pull analysis (Sonis, 1980), based on the results of section 4, consists of the successive extraction from an actual migration stream of the shares corresponding to the constructed set of extreme tendencies. At the beginning, we choose the main extreme tendency; then we construct an extreme migration flow, which is the complete expression of this tendency, and determine its share (weight) in the actual migration and simultaneously determine the residual of the actual migration after the extraction of the action of the main extreme tendency. In this residual, we choose the next extreme tendency, and so forth. The most significant fact is that the set of residuals corresponds to the migrationally meaningful set of the "bottlenecks," corresponding to those parts of the actual migration where the action of migration factors compels the actual migration to diverge from the extreme flow. The appearance of obstacles preventing or supporting the repulsion or attraction from or to some region can be interpreted as the realization of the Stouffer principle of intervening opportunities (Stouffer, 1960). Simultaneously, these migration "bottlenecks" determine the weights of the extreme migration flows M_s in the actual migration stream *M*.

5.2. Normalized space of admissible migration states.

The description of the polyhedron of admissible migration states (5.3) can be simplified by compressing them into a many-dimensional unit cube of Markovian matrices $R = \left[r_{ij} \right] = \left[x_{ij} / K_j \right]$ for Pull analysis

$$r_{ij} \geq 0, \quad i, j = 1, 2, ..., n$$
$$\sum_{i=1}^{n} r_{ij} = 1, \quad j = 1, 2, ..., n \qquad (5.5)$$

The correspondence between the matrices X of admissible migration from the polyhedron (5.3) and the Markovian matrices R from the normalized polyhedron (5.5) is one to one; transfer from matrix R to X is easily done by multiplication of columns of the matrix R on the sums K_j. The unit cube of the Markovian matrices with fixed zero coordinates is generated by the vertices, which are 0-1 Markovian matrices with only one non-zero component 1 in each column. The matrix $M = \left[m_{ij} \right]$ of an actual migration can be converted into the Markovian matrix $R_1 = \left[m_{ij} / K_j \right]$ within the unit cube (5.5) of all Markovian matrices with fixed zero components; thus, the procedure of the decomposition can be applied for Pull analysis of a normalized migration. Moreover, because of the 0-1 structure of the vertices, each extreme tendency can be represented with the help of a map indicating the non-zero normalized flows.

The schemes of Push and Pull analysis include similar numerical procedures. Since *"each main current of migration produces a compensating counter-current"* (Ravenstein, 1885) the results of Push and Pull analysis usually complement each other.

5.3. Example of the decomposition analysis.

As an example, we will consider the repulsion in inter-regional migration of the Israeli population between the three main regions: North, Center and South in 1985. The origin-destination matrix of migration M_{1985} is

M_{1985}	to North	to Center	to South
from North	–	10642	2398
from Center	7630	–	7019
from South	2502	10347	–

with the final distribution of in-migrants $K_1 = 10132, K_2 = 20989, K_3 = 9417$. The actual migration matrix M_{1985} corresponds to the normalized pull migration state $R_1 = \begin{bmatrix} 0 & 0.507 & 0.255 \\ 0.753 & 0 & 0.745 \\ 0.247 & 0.493 & 0 \end{bmatrix}$. Thus state belongs to the unit cube of admissible pull states, which contains all markovian matrices of the form (see figure 1)

$$R = \begin{bmatrix} 0 & r_{12} & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & r_{32} & 0 \end{bmatrix}, r_{21} + r_{31} = r_{12} + r_{32} = r_{13} + r_{23} = 1 \qquad (5.6)$$

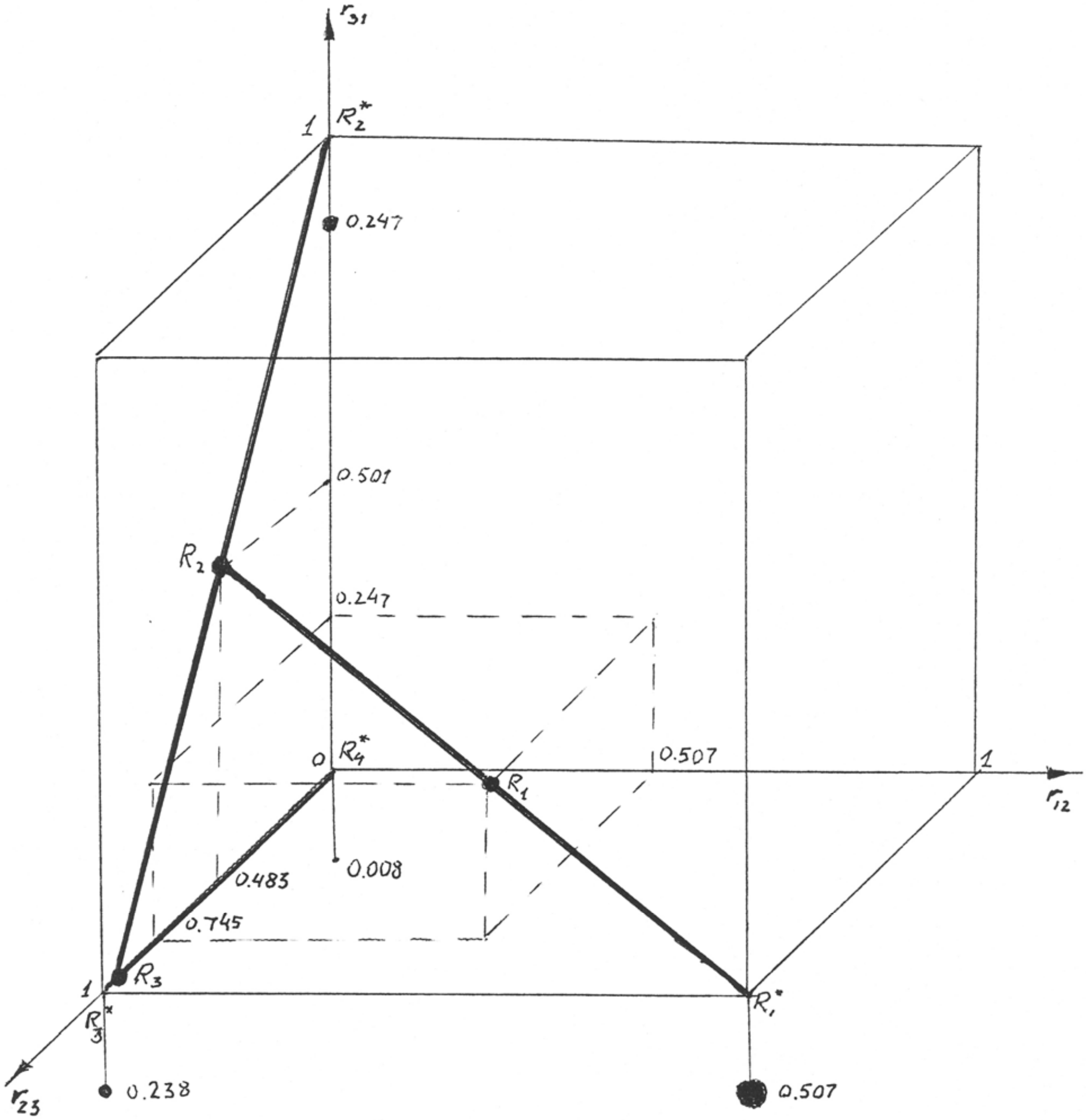

Figure 2. Unite cube of admissible pull states and geometrical visualization of the decomposition algorithm.

The eight vertices of this unit cube, which correspond to the extreme tendencies, have the form:

$$R_1^* = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, R_2^* = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, R_3^* = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, R_4^* = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$
$$R_5^* = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, R_6^* = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, R_7^* = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_8^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad (5.7)$$

The interpretation of the extreme tendencies corresponding to these matrices is as follows: matrices R_1^* and R_3^* are associated with extreme attraction of migrants to Center from North and South and from Center either to North or South; matrices R_3^* and R_8^* are associated with extreme attraction of migrants to South from North and Center and from South either to North or Center: matrices R_5^* and R_7^* are associated with extreme attraction of migrants to North from Center and South and from North either to Center or South; matrices R_4^* and R_6^* are associated with extreme attraction of migrants within circular flows North-South-Center-North clockwise or counterclockwise, respectively. The first extreme tendency R_1^* acting within the actual normalized migration state has a form $R_1^* = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, which represents the attraction from North and South to the Center and the attraction from the Center to the North. The weight of this tendency is 0.507, so the decomposition $R_1 = 0.507 R_1^* + 0.493 R_2$ holds. This implies that $R_2 = 2.028 R_1 - 1.028 R_1^* = \begin{bmatrix} 0 & \bullet & 0.517 \\ 0.499 & 0 & 0.483 \\ 0.501 & 1 & 0 \end{bmatrix}$ where $\bullet$ marks the place of the first "bottleneck" problem – the interdiction to the attraction from Center to North. The extraction of the extreme tendency from the first remainder R_2 gives $R_2 = 0.501 R_2^* + 0.499 R_3$ where the second extreme tendency and the second remainder are $R_2^* = \begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, R_3 = 2.004 R_2 - 1.004 R_2^* = \begin{bmatrix} 0 & \bullet & 0.033 \\ 1 & 0 & 0.967 \\ \bullet & 1 & 0 \end{bmatrix}$. Thus, the second extreme tendency represents the attraction to the South from north and Center and the attraction to North from South. The second remainder includes the action of the next "bottleneck" problem – the interdiction to the attraction from North to South.

The decomposition of the third remainder has a form:

$$R_3 = 0.967R_3^* + 0.033R_4^*$$

where $R_3^* = \begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix}, R_4^* = \begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$.

All this implies that the complete decomposition is:

$$R_1 = 0.507R_1^* + 0.247R_2^* + 0.238R_3^* + 0.008R_4^* =$$

$$= 0.507\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.247\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + 0.238\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix} + 0.008\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}.$$

In this decomposition the most prominent extreme tendency with weight 50.7% (barycentric coordinate $p_1 = 0.507$) corresponds to the tendency of attraction to Center from North and South and from Center to North. The second extreme tendency with weight 24.7% (barycentric coordinate $p_2 = 0.247$) includes the "bottleneck" interdicting to the first tendency by restricting the attraction from Center to North. The third extreme tendency with weight 23.8% (barycentric coordinate $p_3 = 0.238$) includes two "bottlenecks," with the first interdicting the first extreme tendency by restricting the attraction of North from Center and the second interdicting the a second extreme tendency by restricting the attraction of South from Center. These three extreme tendencies explain 99.2% of pull migration phenomenon. The remaining fourth extreme tendency explains only 0.8% of the phenomenon.

5.4. Interconnections between Pull and Push analysis.

If we replace each 0-1 Markovian matrix R_i^* which represents the vertex of the unit cube (5.3) of the normalized admissible pull states by the transposed stochastic matrix S_i^*, we will obtain the vertices of the unit cube of the normalized admissible push states. The relationship between push and pull migration phenomena obtained in the theoretical migration literature in the "Laws of migration" by Ravenstein (1885; see also Lee, 1966) the form of the concept of stream and counter stream: *"for every major migration stream a counter stream develops"*, and *"the efficiency of stream and counter stream (i.e. ratio of stream to counter stream or the net migration generated by the opposite flows) tends to be low if origin and destination are similar"*. For the empirical confirmation of this concept let us compare the normalized pull state $R_1 = \begin{bmatrix} 0 & 0.507 & 0.255 \\ 0.753 & 0 & 0.745 \\ 0.247 & 0.493 & 0 \end{bmatrix}$ and push state $S_1 = \begin{bmatrix} 0 & 0.815 & 0.185 \\ 0.521 & 0 & 0.475 \\ 0.194 & 0806 & 0 \end{bmatrix}$ generated by the same actual migration table M_{1985}.

Their decompositions into the weighted sums of extreme tendencies are:

$$R_1 = 0.507\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.247\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + 0.238\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix} + 0.008\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$$

$$S_1 = 0.521\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 0.284\begin{bmatrix} 0 & 1 & 0 \\ \bullet & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.184\begin{bmatrix} 0 & 0 & 1 \\ \bullet & 0 & 1 \\ 1 & \bullet & 0 \end{bmatrix} + 0.011\begin{bmatrix} 0 & 1 & \bullet \\ \bullet & 0 & 1 \\ 1 & \bullet & 0 \end{bmatrix}.$$

These push and pull decompositions can be interpreted as the spatial symmetry of repulsion and attraction.

5.5. Polyhedral catastrophic dynamics.

If some temporal sequence of migration origin-destination matrices exists for the sequence of different time periods but for the same territorial differentiation of migration origin and destinations, then the corresponding normalized spaces of pull (or push) admissible migration states coincide with the same unit cube (5.3). Hence, the temporal sequence of migration origin-destination matrices generates the movement of the point of the normalized pull (push) migration state $R_1 \; (S_1)$ within the cube of admissible states, and the decomposition of the normalized state generates the simplex whose vertices present the extreme tendencies in the normalized state. The temporal sequence of the migration matrices generates the sequence of simplexes, whose vertices belong to the unit cube of normalized admissible states. This temporal polyhedral dynamics is structurally stable, if the sequence includes only identical simplexes. The dynamics are partially structurally stable if the simplexes include the same partial set of identical vertices, presenting the same set of extreme tendencies. The dynamics are catastrophic if there are no identical subsets of vertices.

As an example, consider the pull polyhedral catastrophic dynamics of the internal migration of Israeli population during the decade 1985-1994:

$$1985 \Rightarrow \begin{bmatrix} 0 & 0.507 & 0.255 \\ 0.753 & 0 & 0.745 \\ 0.247 & 0.493 & 0 \end{bmatrix} = 0.507\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.248\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + 0.238\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix} + 0.008\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$$

$$1986 \Rightarrow \begin{bmatrix} 0 & 0.508 & 0.269 \\ 0.743 & 0 & 0.731 \\ 0.257 & 0.492 & 0 \end{bmatrix} = 0.508\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.249\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + 0.232\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix} + 0.011\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$$

$$1987 \Rightarrow \begin{bmatrix} 0 & 0.504 & 0.255 \\ 0.750 & 0 & 0.745 \\ 0.250 & 0.496 & 0 \end{bmatrix} = 0.504\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.250\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + 0.241\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix} + 0.005\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$$

$$1988 \Rightarrow \begin{bmatrix} 0 & 0.498 & 0.213 \\ 0.761 & 0 & 0.787 \\ 0.239 & 0.502 & 0 \end{bmatrix} = 0.502\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.254\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & \bullet & 0 \end{bmatrix} + 0.212\begin{bmatrix} 0 & 1 & 1 \\ \bullet & 0 & 0 \\ 1 & \bullet & 0 \end{bmatrix} + 0.032\begin{bmatrix} 0 & 1 & \bullet \\ \bullet & 0 & 1 \\ 1 & \bullet & 0 \end{bmatrix}$$

$$1989 \Rightarrow \begin{bmatrix} 0 & 0.483 & 0.243 \\ 0.770 & 0 & 0.757 \\ 0.230 & 0.517 & 0 \end{bmatrix} = 0.517\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.243\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & \bullet & 0 \end{bmatrix} + 0.230\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & \bullet & 0 \end{bmatrix} + 0.010\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ \bullet & \bullet & 0 \end{bmatrix}$$

$$1990 \Rightarrow \begin{bmatrix} 0 & 0.534 & 0.199 \\ 0.824 & 0 & 0.801 \\ 0.176 & 0.466 & 0 \end{bmatrix} = 0.534\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.267\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.173\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & \bullet \\ 1 & 1 & 0 \end{bmatrix} + 0.026\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$$

$$1991 \Rightarrow \begin{bmatrix} 0 & 0.583 & 0.211 \\ 0.846 & 0 & 0.789 \\ 0.154 & 0.417 & 0 \end{bmatrix} = 0.583\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.211\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 0.154\begin{bmatrix} 0 & \bullet & \bullet \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + 0.052\begin{bmatrix} 0 & \bullet & \bullet \\ 1 & 0 & 1 \\ \bullet & 1 & 0 \end{bmatrix}$$

$$1992 \Rightarrow \begin{bmatrix} 0 & 0.592 & 0.223 \\ 0.811 & 0 & 0.777 \\ 0.189 & 0.408 & 0 \end{bmatrix} = 0.592\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.219\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 0.185\begin{bmatrix} 0 & \bullet & 0 \\ \bullet & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + 0.004\begin{bmatrix} 0 & \bullet & 1 \\ \bullet & 0 & \bullet \\ 1 & 1 & 0 \end{bmatrix}$$

$$1993 \Rightarrow \begin{bmatrix} 0 & 0.536 & 0.163 \\ 0.840 & 0 & 0.837 \\ 0.160 & 0.464 & 0 \end{bmatrix} = 0.536\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.301\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.160\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & \bullet \\ 1 & 1 & 0 \end{bmatrix} + 0.003\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}$$

$$1994 \Rightarrow \begin{bmatrix} 0 & 0.500 & 0.185 \\ 0.803 & 0 & 0.815 \\ 0.197 & 0.500 & 0 \end{bmatrix} = 0.500\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.303\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.185\begin{bmatrix} 0 & \bullet & 1 \\ \bullet & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} + 0.012\begin{bmatrix} 0 & \bullet & \bullet \\ \bullet & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

This sequence of pull decompositions includes three years, 1985-1987, of complete structural stability; i.e., in this time interval, all extreme tendencies are repeated and, moreover, their weights (barycentric coordinates) preserve the rank-size ordering.

Nevertheless, the places of bottleneck problems are stable only partially. In the years 1988-1989, the main extreme tendency that is stable in the previous three years is replaced by an extreme tendency that was only third in the previous three decompositions, and the main extreme tendency in the previous three years moved to second place in the next two years. The decomposition simplex, that was stable in 1985-1987, is replaced in 1988-1989 by a decomposition simplex including as vertices the previous main extreme tendencies. In the years 1990-1994, the structural stability of pull decomposition is only partial; the decomposition simplexes in these years include the same main extreme tendency as in 1985-1987. Other tendencies and their corresponding bottlenecks undergo different catastrophic changes.

Consider the average 1985-1994-pull decomposition:

$$\begin{gathered}1985-1994 \Rightarrow \begin{bmatrix} 0 & 0.524 & 0.222 \\ 0.790 & 0 & 0.778 \\ 0.210 & 0.476 & 0 \end{bmatrix} = \\ = 0.524\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + 0.254\begin{bmatrix} 0 & \bullet & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.210\begin{bmatrix} 0 & \bullet & 1 \\ 0 & 0 & \bullet \\ 1 & 1 & 0 \end{bmatrix} + 0.012\begin{bmatrix} 0 & \bullet & 1 \\ 1 & 0 & \bullet \\ \bullet & 1 & 0 \end{bmatrix}\end{gathered} \qquad (5.8)$$

This reveals that the ten-year polyhedral catastrophic dynamics represents the oscillation of the simplexes of the actual normalized pull migration states near the simplex of the average normalized pull migration state.

The geometric picture of migration dynamics presented here reflects the competition and complementarity between different migration tendencies existing simultaneously in space and in time. Each such spatio-temporal tendency has only partial realization in the concrete migration flow because other tendencies constantly competing and interfering into the action of each other tendency. The weight of each tendency is defined by the bottleneck location problems caused by this competition that reflects the migrational intervening opportunities and intervening obstacles.

6. Feedback Loop Decomposition Analysis of spatial economic systems: hierarchy of spatial/functional feedback loop production cycles

The purpose of this section is to reveal the hierarchical structure of economic production cycles within linear economies and spatial production cycles within multi-regional economies. Each production cycle represents the feedback loop relations between the economic sectors and spatial production cycles represent the feedback loops between the activities in different regions. It is important to note that the consideration of the feedback loops in trade theory is a relatively new, substantial event connected with the detailed analysis of the vertical specialization of trade flows (Hummels *et al*., 1998.) The analytical technique employing permutation matrices and the decomposition of flow into feedback loops can be utilized in trade theory (in the form of the linear programming personnel assignment algorithm). Furthermore, the scaling of feedback loops, connected to spatial and functional economic disaggregation of trade, can be interpreted using the Matrioshka principle of hierarchical inclusion of economic activities flows into the spatial loops. The essential consequence of the decomposition of the overall matrix trade flow into the sum of block-permutation matrices of feedback loops provides the possibility to visualize the production cycles with the help of shadowing of different feedback loops in different shades. This visualization represents in fine detail the rich information about spatial and economic interdependencies within interregional trade on the spatial macro-economic level of interregional primary, secondary and tertiary economic activities, sub-activities and individual industries. In such a manner, the table of flows of intermediate goods can be converted into spatial and functional maps of the hierarchy of production cycles. Much more impressive visualization of spatial production cycles can be achieved by using different colors instead of shadowing (see, Sonis, *et al.,* 2001).

9.1. Quasi-permutation matrices and closed feedback loops of the intra-regional production cycles.

Consider a linear economy presented by the matrix of the intra-regional flows of the intermediate goods between n different economic sectors:

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix} \qquad (6.1)$$

The central point of the feedback loops decomposition analysis is the use of functional feedback loops represented by quasi-permutation matrices. By definition, a quasi-permutation matrix includes in each row and column only one non-zero flow.

For example, within a three-sector economic system:

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \qquad (6.2)$$

there are six quasi-permutation sub-matrices

$$\begin{gathered}
M_1 = M_{(1)(2)(3)} = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{pmatrix} \\
M_2 = M_{(132)} = \begin{pmatrix} 0 & 0 & m_{13} \\ m_{21} & 0 & 0 \\ 0 & m_{32} & 0 \end{pmatrix} \\
M_3 = M_{(123)} = \begin{pmatrix} 0 & m_{12} & 0 \\ 0 & 0 & m_{23} \\ m_{31} & 0 & 0 \end{pmatrix}
\end{gathered} \tag{6.3}$$

$$\begin{gathered}
M_4 = M_{(1)(23)} = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & 0 & m_{23} \\ 0 & m_{32} & 0 \end{pmatrix} \\
M_5 = M_{(12)(3)} = \begin{pmatrix} 0 & m_{12} & 0 \\ m_{21} & 0 & 0 \\ 0 & 0 & m_{33} \end{pmatrix} \\
M_6 = M_{(13)(2)} = \begin{pmatrix} 0 & 0 & m_{13} \\ 0 & m_{22} & 0 \\ m_{31} & 0 & 0 \end{pmatrix}
\end{gathered} \tag{6.4}$$

that corresponds to the following permutations of the sequence 1, 2, 3:

$$\begin{gathered}
M_{(1)(2)(3)} \square (1)(2)(3);\; M_{(132)} \square (132);\, M_{(123)} \square (123); \\
M_{(1)(23)} \square (1)(23);\, M_{(12)(3)} \square (12)(3);\, M_{(13)(2)} \square (13)(2)
\end{gathered} \tag{9.5}$$

Here the block-permutation matrices, $M_{(132)}, M_{(123)}$, represent production cycle reflecting the trilateral connections between three sectors, the block-diagonal matrix, $M_{(1)(2)(3)}$, represents the circulation of intermediate flows within the same sector and the quasi-permutations, $M_{(1)(23)}, M_{(12)(3)}, M_{(13)(2)}$, represent the spatial production cycle namely, the bilateral trade between two sectors and the circulation within the remaining sector.

It is important to note that the following decompositions of the matrix, M, in the sum of the quasi-permutation matrices hold:

$$\begin{gathered}
M = M_1 + M_2 + M_3 \\
M = M_4 + M_5 + M_6
\end{gathered} \tag{6.6}$$

The analytical presentation of the decompositions (6.6) can be transformed into graphical representation (see figure 4).

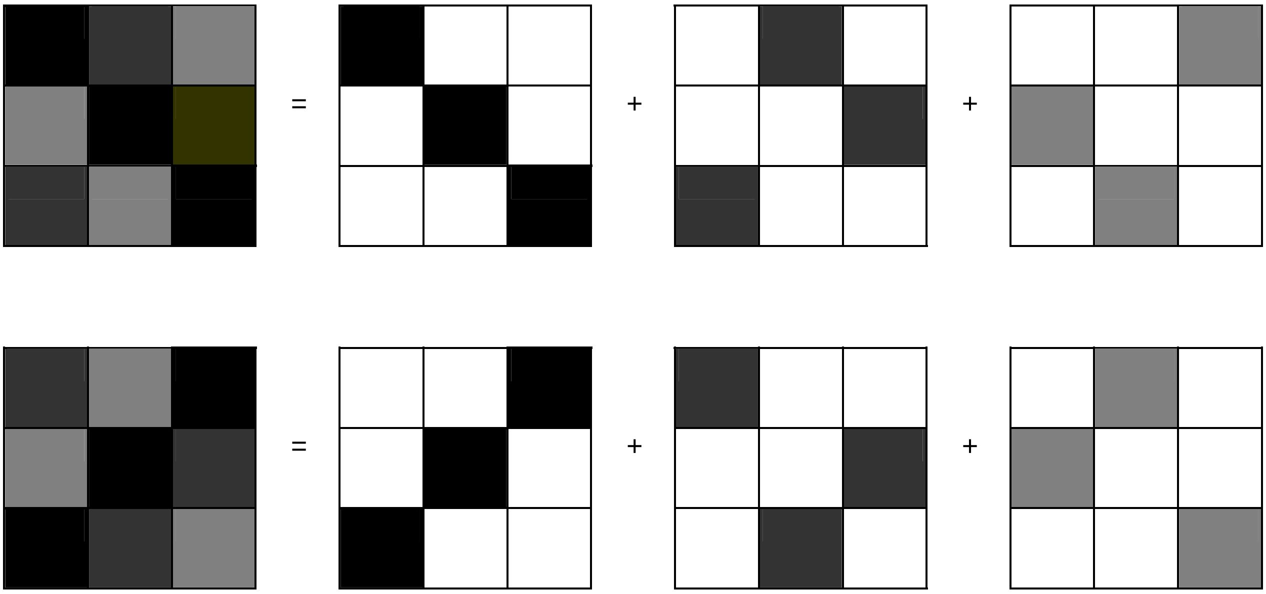

Figure4.. Graphical representations of the feedback loop decompositions (6.6).

In the case of *n* economic sectors, there exists *n!* different quasi-permutation matrices and it is possible to prove that the number of possible additive decompositions of the type (9.6) is equal to *(n-1)!(n-2)!...2!*. Thus, the problem focuses on the appropriate choices from this very large set of possible decompositions, namely, the decompositions with sound economic meaning. Let us stress that we will consider only the complete feedback loops, including all *n* sectors. This assumption does not restrict the generality of our approach, because each non complete feedback loop including $m < n$ sectors can be completed by adding a complimentary feedback loop including the remaining $(n - m)$ sectors.

6.2. Superposition of intra-regional production feedback loop cycles: decomposition algorithm.

One natural method for dealing with a large amount of complete feedback loops is, of course, the derivation of some hierarchical structure. Essentially, the hierarchical feedback loop approach attempts to extract complete feedback loops between different sectors that successively account for the most "explanation" in each stage of the selection process. The procedure continues until all transaction flows have been included. One can define the *flow intensity* of a complete feedback loop (the *flow intensity* of a production cycle) as the sum of all flows of the corresponding quasi-permutation sub-matrix M'. If all non-zero entries of M' are replaced by ones, the result is a so-called *permutation matrix* $P^{/}$. This zero-one matrix corresponds to some permutation of the sequence of numbers *1,2 ,.., n,* which represents the structure of the corresponding complete feedback loop. In our study, the hierarchy of all complete feedback loops is defined as the sequence of quasi-permutation sub-matrices $M^{/}$ chosen according to the rank-size of their flow intensities in such a manner that the matrix of flows of intermediate goods will be decomposed into the sum of quasi-permutation matrices, belonging to this hierarchy. This means that on the top of the hierarchy one finds the complete feedback loop with the maximal flow intensity.

The problem of the determination of the quasi-permutation sub-matrix with the maximal flow intensity is mathematically equivalent to the solution of the Koopmans-Beckman (1957) optimal personnel assignment of *n* persons (here rows) between *n* jobs (here columns) in such a way that one person will have one job while profit is maximized. Here profit is defined by the size of the flows in matrix *M*.

The numerical identification process for the construction of above mentioned hierarchy reflects the following statement: Each economic flow matrix can be presented as a sum of quasi-permutation matrices, presenting the spatial production loops.

$$M = M_{1} + M_{2} + ... + M_{n} \qquad \textbf{(6.7)}$$

The following crude approximate procedure for the derivation of production cycles can be proposed: first, of all in the matrix *M* of all intermediate flows, we will chose the one with the largest flow. This flow will define the first component of the production cycle. Further, we will exclude from matrix *M* the row and column including this component. In the remaining matrix, we will chose the next largest component of the production cycle, etc. After *n*-1 such steps the feedback loop usually will be identified. Unfortunately, this simple procedure does not always yield a closed loop, because there are cases when on some step it will be impossible to find the component for choice. In this case it is necessary to apply the Koopmans-Beckmann algorithm, which usually helps to find the second best component. This second best component in many cases is possible to guess.

6.3. Vertical specialization of production and the economic meaning of the multi-regional aggregated spatial feedback loop production cycles.

It is important to note that above-mentioned procedure of the superposition of feedback loop production cycles can be transferred from the case of a single economy to the case of spatial production cycles within multi-regional economies. This possible to do if the components of the flow matrix (9.1) represent the aggregated economic flows between regions. The justification of consideration of a set of spatial interregional production cycles, presented by the feedback loops of interregional flows of intermediate goods is based on an important phenomenon of vertical specialization which describes the use of imported inputs for producing goods that are exported (see, Bruelhart and Hine, 1999).

Balassa (1967, p.97) coined the term vertical specialization. In the paper (Hummels *et al*, 1998, p.81) the following definition of vertical specialization was introduced and discussed: "(1) a good must be produced in multiple sequential stages, (2) two or more countries must specialize in producing some, but not all, stages, and (3) at least one stage must cross an international border more then once... Thus, countries link sequentially to produce a final good."

The empirical models of interregional trade tend to be either very macro in nature (such as computable general equilibrium models) or they operate at a very micro, sector-by-sector level. Feedback loop analysis occupies an intermediate position between macro analysis of linkages derived from typical input-output analysis and the very micro structural path analysis.

6.4. The Matrioshka imbedding principle for the nested disaggregated hierarchy of spatial feedback loop production cycles.

It is necessary and possible to combine the inter-regional and inter-sectoral interdependencies. To this aim, the aggregated table needs to be replaced by the detailed disaggregated table describing the interplay between the inter-sectoral and inter-regional interdependencies. In such an extension, the analysis will relate to sectors per region. Of course, other levels of aggregation of sectors into primary, secondary and tertiary economic activities, as well as the disaggregation of regions (countries) on sub-regions should be included into the general picture of spatial/functional production cycles feedback loops (see Sonis *et al.*, 1993).

Thus, the problem devolves to one of choosing from this very large set of possible decompositions the decompositions with the sound economic meaning. This problem can be solved through the adoption of a hierarchical stepwise approach; the procedure operates at successive levels in the system, but the approach at each stage is similar to the study of intra-regional production cycles describes in sections 9.1-9.3. This top-down decomposition may be considered analogously to an exfoliation process in the removal of the layers of an onion or to the construction of hierarchy of feedback loops ordered with the help of aggregated cumulative flows within the blocks belonging to the loop. The feedback loops on the inner hierarchical level of economic activities should be placed into the loops of the higher levels in the form of the Matrioshka doll (Russian doll within doll, within doll…) in which successively smaller dolls of exactly the same shape and style are nested within the larger dolls. Hence, the Matrioshka embedding approach examines the intra- and inter-regional transactions in terms of the hierarchical structure of feedback effects drawing upon the superposition principle conceptual framework.

6.5. Spatial Production cycles in the European Common Market, 1965-1980.

This section deals with an example of the complication of the hierarchy of spatial production cycles of the European Common Market, 1965-1980. European economic integration has progressed through different stages, starting with six countries in the 1950s and expanding to twelve countries in the 1980s. The further augmentation of the European Common Market in modern times expands the European Economic Community to the sixteen countries. Lowering and abolishing internal tariffs and non-tariff barriers, establishing common external tariffs, common agricultural and industrial policies and common currency etc., have had a profound effect on economic interaction, both within and between the economies involved. Clearly, there is interplay between the more or less continuous deepening of complexity of the European Community and its discrete extensions. As economies integrate, the spatial structure of economic interaction will change because of trade creation and trade diversion per product group. Subsequent extensions will again change the pattern of spatial economic specialization, (inter) dependence and dominance. An understanding of the intricacies of the process of European economic integration and complication requires attention to be focused on the sectoral and spatial dimension in an integrated framework, such as the set of full intercountry input-output tables for European Community for 1965-1985 (for details, see Schilderinck, 1984; Oosterhaven, 1989; Boomsma *et al*., 1991). It is only with such a data set that one would be able to specify the degree to which the EC economies have changed their dependence on their own national markets vis à vis those elsewhere within the Community and the rest of the world. This section will present the first years of the spatial economic interaction as well as the change over time in the structural pattern of this interaction.

The major element of the Feedback Loop Approach is the identification of a series of (aggregate) spatial production cycles such that each country is allowed precisely one transaction flow entering it and one flow leaving it. A series of spatial production cycles economically represents a chain of multilateral influences, which are based on backward of forward linkages.

6.5.1. Spatial production cycles of the European Community in 1975.

Table 1 presents the 8 x 8 matrix (n = 8) of all aggregate intermediate transaction flows for the nine European Community countries in 1975: West Germany, France, Italy, The Netherlands, Belgium/Luxembourg, United Kingdom, Ireland and Denmark.

Table1.Total intermediate transactions in the European Community in 1975

(in millions of EUR,)

from / to	*GE*	*FR*	*IT*	*NE*	*BL*	*UK*	*IR*	*DK*
GE	313390	6186	4003	5314	3979	2847	103	1233
FR	5491	173278	3146	1500	3191	2621	114	238
IT	3866	2606	125431	671	641	1278	39	155
NE	6471	2022	1177	39890	3091	2555	55	391
BL	4055	3332	788	2942	25424	1163	32	235
UK	1794	1470	795	1265	949	161213	824	641
IR	133	79	56	53	50	1091	5291	7
DK	613	152	196	140	88	592	11	18794

Source: Schilderink, 1984

The complete hierarchy of spatial production cycles can be derived with the help of analytical technique developed in the section 9.4. The results of analysis are summarized in table 2 (see Sonis, Oosterhaven and Hewings, 1993).

Table 2. Rank-size Core-Ring hierarchical structure of feedback loops.

Rank	Structure of complete self-influence feedback loops	Flow intensity	%
Intracountry production cycles			
1	(GE)(FR)(UK)(IT)(NE)(BL)(DK)(IR)	862712	90.7
Core-Ring Intercountry spatial production cycles.			
2	(GE FR BL NE) (IT UK IR DK)	21095	23.8
3	(GE NE BL FR) (IT DK IR UK)	19280	21.8
Bilateral Intercountry spatial production cycles.			
4	(BL GE)(FR IT)(UK DK)(NE IR)	15128	17.1
5	(GE IT)(FR UK)(NE DK)(NE IR)	12573	14.2
6	(GE UK)(NE FR)(BL DK)(IR IT)	8581	9.7
7	(NE UK)(GE DK)(BL IT)(FR IR)	7288	8.2
8	(BL UK)(NE IT)(FR DK)(IR GE)	4586	5.2
			100.0

The transaction flows of the European Community in 1975 reflect, as the major self-influence tendency, the *domestic* circulation of the intermediate flows within the same country. For the *intercountry* flows, the rank-size hierarchy of spatial production cycles appears which reveal two distinct components:

1. A primary Core-Ring spatio-economic structure with a balance in the Push-Pull forces (backward and forward linkages), included in the first two most "loaded" spatial production cycles loops represents 45.5 % of the overall intercountry transactions flows.
2. A secondary spatio-economic structure that consists of five complete feedback loops

with sets of balanced bilateral trade flows.

It is important to underline that bilateral interactions do not appear in the pure form in two largest spatial production cycles. The most loaded bilateral closed subloops, however, are dissolved in these important Core-Ring spatial production cycles.

6.5.2. Catastrophic Structural Dynamics of European production cycles, 1970, 1975 and 1980.

Next, the important question about the temporal stability of the spatio-economic hierarchy found for 1975 arises. For such an analysis, the preliminary data from the European Community intercountry input-output tables for 1970 and 1980 may be used (see Boomsma *et al.*, 1991). These tables, however, are not entirely comparable because of differences in the geographical delimitation (5 countries in 1970, 8 in 1975, 7 in 1980), while the table for 1980 does not cover the entire European Community because Ireland is lacking. Nevertheless, one may compare the main results qualitatively.

In all years, the first spatial production cycle exclusively contains domestic intermediate transactions. Hence, this major feature in constant in time. Table 3 presents in the qualitative form of the second and the third spatial production cycles.

Table 3. Core-Ring production cycles in 1970, 1975 and 1980.

Rank	Year	Structure of production cycles		%
		Core	Ring	
	1970			
2		(BL NE GE FR IT)		31.1
3		(FR GE NE BL IT)		26.7
	1975			
2		(BL NE GE FR)	(IT UK IR DK)	23.8
3		(FR GE NE BL)	(DK IR UK IT)	21.8
	1980			
2		(BL NE GE FR	IT UK DK)	22.8
3		(NE BL FR GE	DK UK IT)	21.7

It is clearly visible from table 3 that the European core, existing in 1970, is consistently preserved in the first two spatial production cycles. In 1975, this core (Germany, France, Belgium/Luxembourg and The Netherlands) is found in the first closed subloops within the complete loops. Moreover, subsequent to the extension of the EC in 1973, an external peripheral ring is developed in 1975 (Italy, United Kingdom, Ireland and Denmark). In 1980 the core and periphery merge into the one complete spatial production cycle. This cycle preserves the order of flows that already occurred in the previous subloops. The merger of Core and Ring most probably is the result of the integration of the new members into the old EC. After their entry in 1973, the removal of several trade tariffs and non-tariff barriers together with their joining the Common Agricultural Policy will have increased their interaction vis à vis the old members; an increase that most probably has been faster than the increase in interaction between the old members themselves.

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