

# Debt and default in a growth model

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## Abstract

This paper presents a small open economy model with capital accumulation and without commitment to repay debt. The optimal debt contract specifies debt relief following bad shocks and debt increase following good shocks and brings first order benefits if the country's borrowing constraint is binding. Countries with less capital (with higher marginal productivity of capital) have a higher debt-GDP ratio, are more likely to default on uncontingent bonds, require higher debt relief after bad shocks and pay a higher spread over treasury. Debt relief prescribed by the optimal contract following the interest rate hikes of 1980-81 is more than half of the debt forgiveness obtained by the main Latin American countries through the Brady agreements.

KEYWORDS: sovereign debt; default; capital flows; optimal contract; world interest rates.

JEL CLASSIFICATION: F3, F4, G1.

## 1 Introduction

Sovereign debt and default have important effects on both economic fluctuations and growth in emerging markets. In terms of economic fluctuations, not only debt crises have large effects on those economies, shocks to the risk of default are also an important source of fluctuations (Neumeyer and Perri (2005), Uribe and Yue (2006)). The growth effects are related to the question of why capital doesn't flow from rich to poor countries, where the marginal productivity of capital is higher (Caselli and Feyrer, 2006). One proposed explanation is that the risk of default prevents larger capital inflows in emerging economies (Reinhart and Rogoff, (2004) Reinhart, Rogoff and Savastano (2003)). Alternative explanations emphasize differences in productivity and question whether marginal productivity of capital are really higher in poor countries (Lucas (1990)).

Yet, those effects do not usually appear in the same model. This paper presents a small open economy growth model in which the country cannot commit to repay debt. Differences between the marginal return to capital in the country and the world interest

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rates provide incentives for borrowing. If the country repudiates its debt, it is excluded from capital markets and incurs an output loss — those are the incentives for repayment.

A country that repudiates its debt faces the threat of sanctions such as loss of access to short term trade credits and seizure of assets.<sup>1</sup> Moreover, in today’s globalized world, debt repudiation would undermine a country’s capacity to obtain mutually beneficial deals in multi-lateral organizations (e.g., WTO). I consider such costs as the key motivation for a country to repay its debt and model them as an output loss. In reality, however, observed punishment is not very harsh and ends up not lasting long, because creditors and debtors have incentives to renegotiate and because default is usually “excusable” — related to bad states of nature. In the model, the output loss due to debt repudiation is not paid in equilibrium, but plays a key role in the analysis. Instead of analyzing renegotiation, I derive the optimal debt contracts. In equilibrium, it is never optimal to break the contract but debt relief (or “partial default”) is sometimes optimal.

A usual problem with models with endogenous decision of debt repayment and capital accumulation is the loss of tractability. Here, I derive some analytical results using first order approximations of the Bellman equations and analyze numerical examples.

In the standard Ramsey model, the central planner solution and the decentralized equilibrium are the same. That is not true in this model, however, because of an externality of capital accumulation. I analyze the central planner solution — it could be decentralized if the government set the appropriate taxes.

I start by analyzing a deterministic model. Countries borrow in order to converge faster to its steady state level of capital and, as there is no uncertainty, there is no default in equilibrium. The output loss incurred in case of default plays a key role in the determination of the incentive compatible level of debt. In equilibrium: (i) debt as a fraction of GDP decreases as the country converges to its steady state level of capital; (ii) countries with binding borrowing constraints (that is, with marginal productivity of capital higher than the risk-free world interest rate) present current account deficits but trade balance surpluses so, as a consequence, (iii) an indebted country will take *more* time to converge to its steady state level of capital than a country in autarky with the same level of capital.

Then, I include uncertainty in the model and analyze the equilibrium in case of uncontingent contracts in order to get intuition and to compare the results with the literature. In equilibrium, (i) countries with less capital (poorer) are more likely to run the risk of defaulting because their marginal productivity of capital is higher, so it pays

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<sup>1</sup>See Bulow and Rogoff (1989a) for a detailed analysis of such costs.

off for them to pay an interest rate premium to obtain extra borrowing; (ii) the level of debt and frequency of default are substantially higher than in endowment economies (with same parameters); (iii) default occurs following bad shocks and (iv) reasonable fluctuations in world interest rates generate substantially more frequent default than reasonable fluctuations in technology.

A problem with the model above is that, following a bad shock, creditors and lenders would have incentives to renegotiate: the creditors would be willing to decrease the value of the debt, making it low enough to be incentive compatible for the country to repay. Thus the possibility of such partial default (or debt relief) would be priced in the bonds. So, contracts would be contingent *de facto* even if not explicitly so.

Next, I analyze the model with contracts contingent on world interest rates and find the optimal debt contract. The contract specifies debt relief following bad shocks and debt increase following good shocks.

The optimal contract has not only second moment benefits but, more importantly, brings first order efficiency gains if the country's borrowing constraint is binding. By transferring debt from the low state to the high state, the optimal contract allows the country to borrow an extra amount without the risk of default (and, therefore, without paying a risk premium). If the country's marginal productivity of capital is bigger than the world interest rates, that increases world output. For example, suppose that it is incentive compatible for the country to repay up to 12 dollars of debt if the state turns out to be high next period and only up to 10 dollars if the state is low. With uncontingent contracts, the country has the choice between borrowing 10 dollars with no risk premium or 12 dollars paying a risk premium. With contingent contracts, the country can borrow more than 10 dollars, but repayment conditional on low state is 10 and on high state is 12.

In the optimal contract, debt relief depends on (i) magnitude of shocks, (ii) persistence of states and (iii) the amount of capital of the country. The effect of level of capital on debt relief is ambiguous but, according to numerical simulations, poorer countries get (slightly) higher debt relief. The output cost has no first order effects on the fraction of debt relief, it just affects the level of debt.

The next step is to compare the debt relief according to the model and the observed debt forgiveness of Latin American debt that resulted from the Brady agreements. The optimal debt contract prescribes substantial debt relief following the increase in US interest rates in the beginning the 80's — more than half of the debt relief observed after some years from the Brady plan agreements. I take it as an evidence that the model can

generate realistic predictions on debt relief — and such contingencies would have been implicitly considered by lenders and borrowers.

However, such debt relief came with a 10-year delay. Thus, a contingent contract could have avoided 10 years of IMF missions, Baker-Brady plans and debt renegotiation. This bargaining process is very costly. The paper argues in favor of writing such contingent contracts even if that has been implicitly done. Many problems related to GDP-indexed debt (moral hazard, measurement issues, positive correlation between GDP's) don't apply to contract contingent on world interest rates. I discuss some of those issues before concluding.

## 2 Related literature

The threat of sanctions is also the key motivation for repaying debt in the model of Bulow and Rogoff (1989a).<sup>2</sup> They analyze the bargaining problem between creditor and lenders and, in equilibrium, renegotiation implies that sanctions are not applied — but are nevertheless important to determine the outcome of the model, as in this paper. Renegotiation is also studied by Fernandez and Rosenthal (1990) and Yue (2005).

Sovereign debt was also analyzed as an (implicitly) contingent claim by Grossman and Van Huyck (1988) and Atkeson (1991). Grossman and Van Huyck (1988) show that an equilibrium in which “excusable” default is allowed without sanctions can be sustained — such contingencies would be implicitly considered. Alfaro and Kanczuk (2005) quantitative analysis builds on Grossman and Van Huyck (1988). Here, I do not model renegotiation or the reasons for why the implicit claims would be honored: I derive the optimal debt contract assuming that sanctions will be applied if and only if the contract is not honored.

A recent literature, building on Eaton and Gersovitz (1981), has been studying the relation between default risk and macroeconomic variables in endowment economies. Arellano (2005) studies an endowment economy in which borrowing occurs mainly because the local government is very impatient, but also to smooth consumption. The government repays its debt if it is incentive compatible to do so, as in this paper. In Arellano (2005), debt is not contingent and the output cost in case of default is paid at some states in equilibrium, so the interpretation of the output cost is not the same as in this paper. In section 4, I study the case of uncontingent debt so the results of this paper can be related to Arellano's contribution (and also to Aguiar and Gopinath (2006)

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<sup>2</sup>See also Bulow and Rogoff (1989b).

and Yue (2005)).

Cohen and Sachs (1986) present a growth model in which debt is repaid only if it is incentive compatible to do so, but assume a linear production function and have no uncertainty. They also analyze a numerical example with decreasing returns to scale — which is basically the same model I have at section 3. Kehoe and Perri (2004) theoretical contribution also builds on a growth model. Wright (2002) is one of the few other examples of models in this literature with capital accumulation — with linear production function. Decreasing returns to capital is one important feature of this paper.

The distinction between the central planner solution and the decentralized allocation is analyzed by Kehoe and Perri (2004). They show that the central planner solution can be decentralized if the central government is in charge of deciding about defaulting or not and taxes capital income to counteract an externality of capital accumulation.

The output cost of debt repudiation, as modelled here, is present in Cohen and Sachs (1986) and in many recent dynamic stochastic models on debt and default (Arellano (2005), Wright (2002), Aguiar and Gopinath (2006)) and that is key for obtaining default following bad shocks.

### 3 Deterministic Model

Consider an open economy that can borrow from abroad but cannot commit to repay its creditors. There is one good, time is discrete, agents live forever and have utility function given by  $\sum_{t=0}^{\infty} \beta^t u(c_t)$ , where  $\beta \in (0, 1)$  is the time discount factor,  $c_t$  is consumption at time  $t$  and  $u(\cdot)$  satisfies the Inada conditions. The good can be used for consumption or transformed into capital. Production ( $y$ ) is given by  $y_t = A.f(k_t)$ ,  $k_t$  is the stock of capital and  $f(\cdot)$  satisfies the Inada conditions. The country can only borrow by issuing one-period debt. If the country defaults in its debt (denoted by  $d_t$ ), productivity falls by a factor of  $\gamma$  — the production function from time  $t$  on is given by  $y_t = (1 - \gamma)A.f(k_t)$  — and the country loses access to capital markets forever.

There is a continuum of risk-neutral lenders that, in equilibrium, lend to the country as long as the expected return on their assets is not lower than the risk-free interest rate in international markets,  $r^*$ . The price of a bond that delivers one unit of good next period with certainty,  $(1 + r^*)^{-1}$ , will be denoted  $q^*$  and  $q^* = \beta$ . There is a maximum amount of debt the country can contract that prevents it from running Ponzi schemes but is not binding otherwise.

The central planner maximizes:

$$\sum_{t=0}^{\infty} \beta^t \cdot u(c_t)$$

subject to:

$$c_t + k_{t+1} = y_t + (1 - \delta)k_t - d_t + q \cdot d_{t+1}$$

where  $q$  is the (endogenously determined) price of debt and  $\delta$  is the depreciation rate.

If the country hasn't ever defaulted and chooses to repay its debt at time  $t$ , the value function is given by:

$$V_{pay}(k, d) = \max_{k', d'} \{u(Af(k) + (1 - \delta)k - k' - d + qd') + \beta V(k', d')\}$$

In case of default, the value function is given by:

$$V_{def}(k) = \max_{k'} \{u((1 - \gamma)Af(k) + (1 - \delta)k - k') + \beta V_{def}(k')\}$$

and  $V(k, d) = \max \{V_{pay}(k, d), V_{def}(k)\}$ .  $V_{def}(k)$  will sometimes be denoted  $V_{def}(k, \gamma)$ .

I assume that decisions about  $k'$  and  $d'$  are made simultaneously and lenders can observe  $k'$  before taking their lending decisions (or condition their decisions on  $k'$ ). As noted by Cohen and Sachs (1986), the country could otherwise have incentives to borrow  $d'$  but then invest less, consume more and default in its debt.<sup>3</sup> This moral hazard problem is studied by Atkeson (1991).

In equilibrium:

1. There is never default. As there is no uncertainty,  $q = q^*$  if the country will repay and  $q = 0$  otherwise. By setting  $d = 0$ , the country can do strictly better than defaulting because that yields the same amount of consumption today and more production next period (by avoiding the  $\gamma Ak^\alpha$  output loss). So, in equilibrium,  $V_{pay}(k, d) \geq V_{def}(k)$ . This constraint is crucial to determine how much debt the economy can accumulate.
2. The maximum level of debt,  $d^{\max}(k)$ , is increasing in  $\gamma$ . By differentiating the value function, we obtain that  $V_{pay}$  is decreasing in  $d$  and  $V_{def}$  is decreasing in  $\gamma$ . It is incentive compatible to repay whenever  $V_{pay}(k, d) \geq V_{def}(k, \gamma)$ , which yields the result.

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<sup>3</sup>To see this, note that in the optimal plan  $V_{pay}(k', d') = V_{def}(k', \gamma)$  and  $u'(c) = \beta \frac{\partial V_{pay}}{\partial k'}(k', d')$ . But  $\frac{\partial V_{pay}}{\partial k'}(k', d') > \frac{\partial V_{def}}{\partial k'}(k')$ , so if the country has already borrowed  $d'$  and hasn't committed to  $k'$ , a marginal decrease in  $k'$  leads to an increase in today's utility that is bigger than the decrease in tomorrow's value.

3. If  $k'$  is below the steady state level of capital ( $k^*$ ),  $d = d^{\max}$ . In steady state,  $k' = k$  and the marginal productivity of capital,  $mpk = Af'(k) - \delta$ , equals the marginal cost of renting an extra unit of capital,  $r^*$ . In this case, the country has no incentive to change the level of its debt, because capital is at the optimal level and smooth consumption can be achieved by always choosing  $d' = d$ . But if  $k < k^*$ ,  $mpk > r^*$  and  $d$  cannot be smaller than  $d^{\max}$ , otherwise the no-default condition would not be binding, so the country could borrow an extra unit at  $r^*$ , invest it and obtain a return bigger than  $r^*$  next period.
4. If  $\gamma = 0$ , no debt can be sustained in equilibrium. In steady state, the economy has no incentive to repay, so one period before it cannot borrow, therefore it also has no incentive to repay its debt one period before it reaches the steady state equilibrium, and by backward induction there can't ever be any debt. It has already been shown in the literature that, with uncertainty, there may be debt in equilibrium even in the absence of output costs (see Arellano, 2005).
5. If  $\gamma = 1$ , the equilibrium corresponds to the full commitment case. Full commitment equilibrium is first best, so the only question is whether it is incentive compatible. If  $\gamma = 1$  it is: with no default, in steady state (that is achieved in one period), the country obtains consumption equal to  $f(k^*) - \delta k^* - r^* d^*$  which is positive because  $d^* \leq k^*$ ,  $f'(k^*) - \delta = r^*$  and  $f(k^*) > k^* f'(k^*)$ . It has already been shown in the literature that with uncertainty and incomplete asset markets, the full commitment equilibrium may not be the first best equilibrium because the possibility of default makes debt somewhat contingent (see Zame, 1993).

Now, consider  $k'_p$  and  $d'$  such that  $V_{pay}(k'_p, d') = V_{def}(k'_p)$  and  $k'_p \leq k^*$ . Then, there exists some  $k \leq k'_p$  and  $d$  such that the country is indifferent between “repaying and choosing  $(k'_p, d')$ ” and “defaulting and choosing  $(k'_d)$ ”. The value functions are:

$$V_{pay}(k, d) = \max_{k'_p, d'} \{u(y + (1 - \delta)k - k'_p - d + qd') + \beta V_{pay}(k'_p, d')\}$$

and

$$V_{def}(k, \gamma) = \max_{k'_d} \{u(y + (1 - \delta)k - k'_d - \gamma y) + \beta V_{def}(k'_d, \gamma)\}$$

$V_{def}$  is defined as a function of  $k$  and  $\gamma$ . If  $\gamma = 0$  and  $d = 0$ , the value functions in case of default and repayment are the same. Define

$$\begin{aligned} V_0(k) &= \max_{k'} \{u(y + (1 - \delta)k - k') + \beta V_{def}(k', 0)\} \\ &= \max_{k'} \{u(y + (1 - \delta)k - k') + \beta V_{pay}(k', 0)\} \end{aligned}$$

By taking a linear approximation of  $V_{pay}(k, d)$  and  $V_{def}(k, \gamma)$  around  $V_0(k)$  and manipulating the linearized expressions, we obtain the results stated below.<sup>4</sup>

**Proposition 1** *In this economy, for a very small  $\gamma$ :*

1. For  $y < y^*$ :

$$\gamma \cdot y_t = (1 - q)d_{t+1} - \Delta d_t$$

and:

$$d_t = q \cdot d_{t+1} + (1 - q) \frac{\gamma y_t}{1 - q}$$

2. In steady state:

$$\gamma \cdot y^* = (1 - q)d^*$$

so:

$$d^* = \frac{\gamma y^*}{1 - q}$$

*Proof: see appendix.*

Part (2) of the proposition shows that in steady state, the country keeps repaying its debt if the interest payment ( $d^* \cdot (1 - q^*)$ ) is not bigger than the output loss ( $\gamma \cdot y^*$ ) due to default. The maximum debt as proportion of GDP is (as a first order approximation) equal to  $\gamma / (1 - q)$ . Positive debt with no uncertainty arises in equilibrium to finance convergence.

The maximum level of debt is proportional to the output loss and inversely proportional to the risk-free interest rate. Note that a change in interest rate from 1% to 2% has the same impact on  $d^{\max}$  as a decrease of 50% in GDP. Fluctuations in interest rates from 1% to 2% are much more common than a 50% fall in output and the next section explores the effects of interest rate and technology fluctuations in a stochastic version of this model.

Part (1) of the proposition shows that for  $y < y^*$ , the condition for default boils down to a comparison between output losses and resources paid to foreign agents *at the present period*. But the increase in debt is endogenously determined considering that the country will be indifferent next period between repaying and not — so, ultimately, debt as period  $t$  is obtained by backward induction from the steady state level of debt.

For  $y < y^*$ , debt is a higher proportion of GDP — the maximum debt at time  $t$  can be written as a convex combination of  $d_{t+1}$  and  $\gamma y_t / (1 - q)$ , and the weight on the former

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<sup>4</sup>The optimal choice of  $k'$  depends on whether the country chooses to default or not. If  $\gamma$  is very small, so is the level of debt, and the second order effects of the changes in  $k$  are negligible, which allows us to obtain the analytical results.

is much bigger. Before reaching the steady state, debt is increasing (in absolute terms) because the output losses due to default are proportional to  $y$ . So, the amount of money paid to foreign countries is smaller than the interest bill —  $(1 - q)d_{t+1}$ . Therefore, it is incentive compatible to repay a (relatively) higher level of debt.

In equilibrium, the country must experience net outflows of resources on the way to convergence. Debt is increasing (financial account is in surplus) but such increase is smaller than the interest paid in its debt. So, current account is in deficit, but the country is a net exporter of goods.

The results do not depend on functional forms of the utility function or the production function, those would only have second order effects.

Other papers in this literature consider that, in case of default, productivity would fall from period  $t + 1$  on and that the fall in productivity would be temporary, not permanent. Proposition ??, presented in the appendix, deals with such cases, in steady state. It also shows that the maximum level of debt obtained at proposition 1 is actually an upper bound (a reasonably tight upper bound, as shown below).

### 3.1 Numerical solution

In the numerical examples of this paper, production and utility function are given by:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad , \quad f(k) = k^\alpha$$

The numerical solution is obtained through value function iteration with the aid of the computer. The state space is discretized, grids of points for  $d$  and  $k$  are constructed and the planner is forced to choose one of the points in the grid.

Calibration: one period correspond to one year.  $A = 1$ ,  $\alpha = 0.36$ ,  $\beta = 1.03^{-1}$ ,  $\sigma = 3$  and  $\delta = 0.10$ . The price of a riskless bond,  $q^*$ , equals  $\beta$ . The productivity loss in terms of default,  $\gamma = 0.01$ . The grids are so that  $k \in \{0.50, 0.51, 0.52, \dots, 5.00\}$  and  $d \in \{0.00, 0.01, 0.02, \dots, 0.62\}$ .

The formulas derived above and the numerical examples yield very similar results. Figure 1 shows the path of  $d$  in the numerical example and as given by the above formula (using the path of  $y$  given by the numerical example). In the numerical examples,  $100 \times d$  has to be an integer. The numbers obtained using the linear approximation, truncated in order to comply with that restriction, are very similar to the numerical results.

Figure 2 shows the behavior of capital, its marginal productivity, capital inflows and debt (as proportion of GDP) in this economy, compared to the closed-economy case and

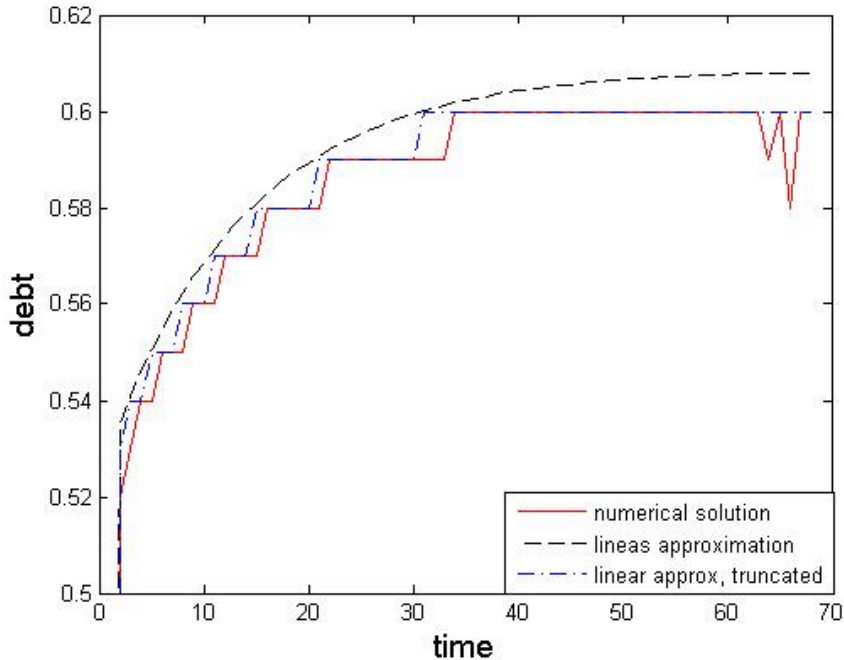


Figure 1:

the full-commitment open-economy case. Without the possibility of default, the level of capital jumps to its steady state level and the marginal productivity of capital equals  $r^*$  in one period. The possibility of default makes convergence slower. Due to the initial capital inflow, the level of capital is higher in this economy than in the closed economy case until they converge (MPK is smaller), but the closed economy slowly catches up, as the open economy will be experiencing net capital outflows during the whole history. Debt stabilizes at 35% of GDP but reaches 50% of GDP at earlier stages.

A usual intuition is that financially open economies should converge faster to its steady state (see, e.g., Barro, Mankiw and Sala-i-Martin, 1995). But that is not what happens here in equilibrium because after the initial (massive) capital inflow, the country experiences net outflows of resources (positive trade balance). An indebted open economy would take *more* time to converge than a closed economy with the same level of capital. An open economy with zero debt would also not converge significantly faster than the closed economy but, before reaching the steady state, would have higher output. In order to experience faster convergence, emerging economies need trade deficits and, as proposition 1 shows, that does not occur in equilibrium.<sup>5</sup>

<sup>5</sup>All of the main South American countries currently have trade balance surpluses — and a big chunk of those surpluses goes back to developed countries as interest payments and profits. On the other hand, most of the Eastern European

Two important ingredients for a more detailed analysis on this issue are absent here: adjustment costs for capital and growth in  $A$  (human capital), but I believe that the framework presented here could be enhanced with those features.

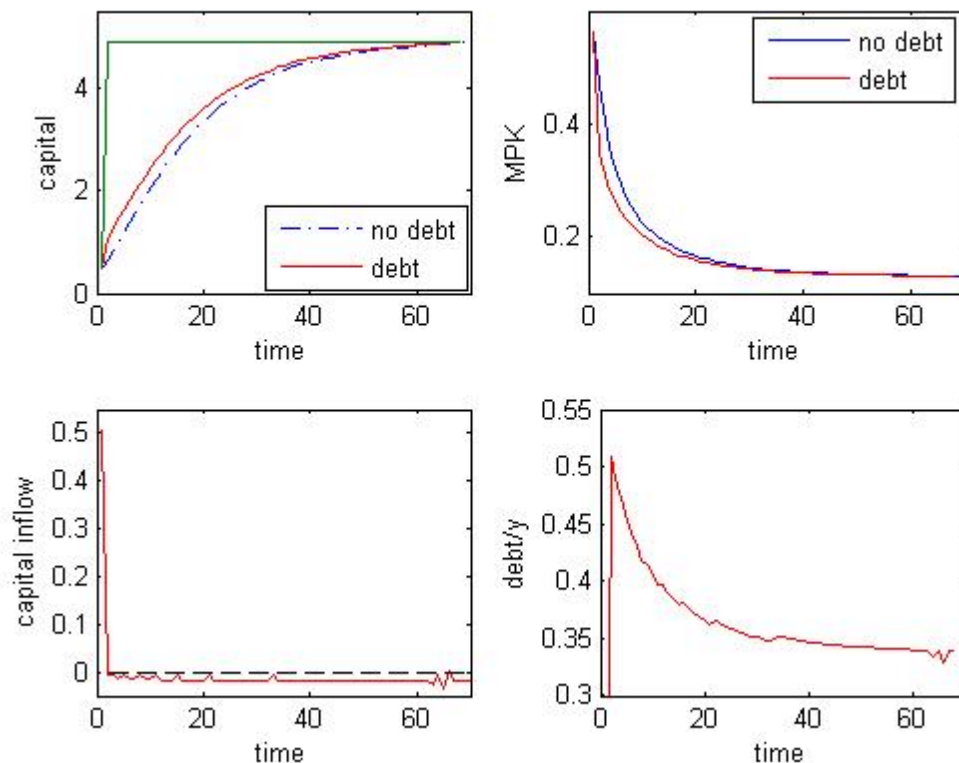


Figure 2: Convergence

If the economy starts at a point with a higher level of capital, the path from then on is pretty much the same as in figure 2 — note that the level of debt is obtained by backward induction from its steady state level.

Reinhart and Rogoff (2004) and Reinhart, Rogoff and Savastano (2003) argue that short-sighted governments would contract excessive levels of debt which would lead to crises and defaults. They blame debt intolerance for capital not flowing from rich to poor countries. Caselli and Feyrer (2006) show that differences in marginal productivity of capital across countries are not that big, but poor countries do have a bit higher marginal productivity of capital. This could be a first step towards a framework to evaluate numerically how much debt is too much and how much of the difference in

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countries have been experiencing positive net capital inflows (trade balance deficits). If the Eastern European countries are still receiving their “initial” capital inflow after opening up to the west, that is what we would indeed expect.

marginal productivity of capital can be explained by capital market imperfections.

Other papers (Arellano (2005), Aguiar and Gopinath (2006) for example) have tried to match empirical facts about debt and default. Assuming very low values of  $\beta$  (below 95% for a quarter), they still predict substantially lower levels of debt than we observe in reality. Those papers study endowment economies that issue debt to smooth consumption. Here, we can see that if debt is issued to buy capital and finance convergence, substantially higher levels of debt are obtained.

## 4 Stochastic model — uncontingent debt

This section consider two stochastic versions of the model, one with stochastic  $A$  and another with stochastic  $q^*$ . There are 2 possible states,  $s_t \in \{h, l\}$ , and the probability of switching to the other state is  $\psi$ :  $\Pr(s_t = h | s_{t-1} = l) = \Pr(s_t = l | s_{t-1} = h) = \psi$ ,  $\psi < 0.5$ . As before, the country can issue only one-period uncounting debt.

### 4.1 Setup

#### 4.1.1 Stochastic technology

Technology is  $A^h$  in the high state and  $A^l$  in the low state,  $A^h > A^l$ . If the country repays, the value function at state  $i$  is given by:

$$V_{pay}^i(k, d) = \max_{k', d'} \left\{ \begin{array}{l} u(A^i f(k) + (1 - \delta)k - k' - d + q^i(k', d')d') \\ + \beta [(1 - \psi)V^i(k', d') + \psi V^j(k', d')] \end{array} \right\}$$

where  $j$  is the other state and  $V^i(k, d) = \max \{V_{pay}^i(k, d), V_{def}^i(k)\}$ . The price of the bond is given by the zero-expected-return condition for the foreign risk neutral agents.

In case of default, the value function is:

$$V_{def}^i(k) = \max_{k'} \left\{ u((1 - \gamma)A^i f(k) + (1 - \delta)k - k') + \beta [(1 - \psi)V_{def}^i(k') + \psi V_{def}^j(k')] \right\}$$

#### 4.1.2 Stochastic foreign interest rate

The price of a riskless bond in international markets is  $q^{*h}$  in the high state and  $q^{*l}$  in the low state,  $q^{*h} > q^{*l}$ . If the country repays, the value function at state  $i$  is given by:

$$V_{pay}^i(k, d) = \max_{k', d'} \left\{ \begin{array}{l} u(Af(k) + (1 - \delta)k - k' - d + q^i(k', d')d') \\ + \beta [(1 - \psi)V^i(k', d') + \psi V^j(k', d')] \end{array} \right\}$$

where  $j$  is the other state and  $V^i(k, d) = \max \{V_{pay}^i(k, d), V_{def}^i(k)\}$ . The price of the bond is given by the zero-expected-return condition for the foreign risk neutral agents: in the low state, expected return must be  $1/q^{*l}$ , in the high state it suffices to be  $1/q^{*h}$ .

In case of default, the value function in both states is:

$$V_{def}(k) = \max_{k'} \{u((1 - \gamma)Af(k) + (1 - \delta)k - k') + \beta V_{def}(k')\}$$

It makes no difference whether foreign interest rates are low or high if the country is excluded from financial markets.

## 4.2 Results

The maximum incentive-compatible level of debt is higher at high states than at low states. Therefore, at high states, the country has 2 options: either (i) it respects the low-state constraint, so interest rate is  $q^*$  and there is no default next period regardless of  $s'$ ; or (ii) it borrows up to its high-state constraint and defaults next period if the state switches to low — in this case the price of bond is  $(1 - \psi)q^*$ , because with probability  $\psi$  the creditor won't be repaid. So the question is: when is it worth for the country to pay an interest premium in order to borrow more?

The bigger is the difference between  $q^{*h}$  and  $q^{*l}$  (or  $A^h$  and  $A^l$ ), the larger is the difference between the borrowing constraint in both states, so the higher are the benefits of paying an interest premium (on the whole debt) to borrow extra. Also,  $\psi$  positively affects the interest premium and negatively affects the difference between  $V^h$  and  $V^l$ , so lower values of  $\psi$  bring more incentives to borrow beyond the low-state limit. If  $\psi$  is arbitrarily small, the country will always choose to borrow up to the high-state limit (while  $s = h$ ) and default when the state shifts to low.

For which values of  $\psi$  can the country choose  $k$  and  $d$  such that there is default if the state switches to low? The intention here is not to contrast the results to real data, but to understand what makes fluctuations in the incentive-compatible level of debt wide enough to generate default. The numerical exercises show that:

1. Default is more likely to occur for low levels of capital: if state is high, the higher is the marginal productivity of capital, the bigger are the benefits of paying an interest rate premium to borrow beyond the bounds consistent with repayment at the low state. So, if  $k$  is smaller, a country is more likely to put itself in position to default.
2. reasonable interest rates fluctuations generate substantially more frequent default than reasonable fluctuations in technology.

Calibration: one period correspond to one year. As before,  $\alpha = 0.36$ ,  $\beta = 1.03^{-1}$ ,  $\gamma = 0.01$ ,  $\sigma = 3$  and  $\delta = 0.10$ . In this model, the country would immediately respond to state changes with dramatic shifts in capital. In the real world, adjustment takes a bit more time. To ease computational time, I constrained net investment to be in the  $[-\delta k, \delta k]$  interval — that is, the net change in capital in one year cannot be bigger than 10% (in absolute value). That seems also more realistic, but I expect results to be similar if no constrain is imposed.

In the model with stochastic interest rates,  $A = 1$ , and  $q^*$  fluctuates around  $\beta$ :  $q^{*l} = 1.04^{-1}$  and  $q^{*h} = 1.02^{-1}$ . With stochastic technology,  $q^* = \beta$  and  $A$  fluctuates around 1:  $A^h = 1.05$  and  $A^l = 0.95$ .<sup>6</sup> The fluctuations in technology are large: technology in the high state is 10% higher than in the low state. On the other hand, world interest rates fluctuations are moderate, between 2% and 4% a year.

Yet, such fluctuations in world interest rates generate three times more default than fluctuations in technology. For example, there is default for  $k \leq 4.62$  if  $\psi = 0.5\%$  in the case of stochastic interest rates, and for  $\psi = 0.15\%$  in the case of stochastic technology; there is default for  $k \leq 4.32$  if  $\psi = 0.7\%$  in the case of stochastic interest rates, and for  $\psi = 0.25\%$  in the case of stochastic technology.

Models trying to match empirical data usually consider fluctuations in endowment and end up generate very little default. Allowing for fluctuations in world interest rates could help matching the facts. The magnitudes of default obtained in this exercise are small compared to real world values, but here we are ruling out partial default — which makes an important difference, as will be clear below.

## 5 Debt contingent on world interest rates

This section analyzes the optimal debt contract for an economy with fluctuations in world interest rates,  $q^*$ . The technology level,  $A$ , is fixed. The debt contract specifies the country's debt at each possible state, and the country has the choice between honoring its debt and walking out. If its borrowing constraint is binding, the country wants to borrow as much as it can and, in order to expand its borrowing possibilities, it chooses to make debt in each of future states as high as possible, respecting the incentive compatibility constraint.

The predictions of the model are contrasted with data from the Latin American debt

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<sup>6</sup>In the case of stochastic interest rates, the grids are:  $k \in \{3.70, 3.72, 3.74, \dots, 6.50\}$  and  $d \in \{-0.50, -0.48, -0.46, \dots, 1.00\}$ . In the case of stochastic technology, it is enough to use:  $k \in \{4.00, 4.02, 4.04, \dots, 5.50\}$  and  $d \in \{-0.30, -0.28, -0.26, \dots, 0.70\}$ .

crisis of the 80's, and the interest rate shock can account for a large part of the observed debt relief.

### 5.1 The optimal debt contract

A risk-neutral creditor that lends  $q^*d'$  must get an expected repayment equal to  $d'$ . Denote by  $d^h$  and  $d^l$  the repayment conditional on high and low state, respectively, and  $\Delta d = d^h - d^l$ . If  $s_{t-1} = h$ , a country that borrowed  $q^{*h}d'$  has debt  $d^h$  if  $s_t = h$  and  $d^l$  if  $s_t = l$  such that  $d^h(1 - \psi) + d^l\psi = d'$ . In equilibrium,  $d^h = d' + \psi\Delta d$  and  $d^l = d' - (1 - \psi)\Delta d$ . If  $s_{t-1} = l$ , a country borrowing  $q^{*l}d'$  has debt  $d^h$  if  $s_t = h$  and  $d^l$  if  $s_t = l$  such that  $d^l(1 - \psi) + d^h\psi = d'$ . In equilibrium,  $d^l = d' - \psi\Delta d$  and  $d^h = d' + (1 - \psi)\Delta d$ . Thus the country is choosing, at every state, one extra variable,  $\Delta d$ . The value functions conditional on repayment are:

$$\begin{aligned} V_{pay}^h(k, d) &= \max_{k', d', \Delta d} \left\{ \begin{array}{l} u(Af(k) + (1 - \delta)k - k' - d + q^h(k', d')d') \\ +\beta [(1 - \psi)V^h(k', d' + \psi\Delta d) + \psi V^l(k', d' - (1 - \psi)\Delta d)] \end{array} \right\} \\ V_{pay}^l(k, d) &= \max_{k', d', \Delta d} \left\{ \begin{array}{l} u(Af(k) + (1 - \delta)k - k' - d + q^l(k', d')d') \\ +\beta [(1 - \psi)V^l(k', d' - \psi\Delta d) + \psi V^h(k', d' + (1 - \psi)\Delta d)] \end{array} \right\} \end{aligned}$$

If contracts can be written and enforced contingent on  $q^*$  (the only source of uncertainty in this economy), debt repudiation is never optimal, because a contract specifying no debt in one of the states is strictly better than repudiating debt at that state: it prevents output losses and changes nothing else, as the interest rate premium due to the possibility of default will be equal to the extra payment in the state that yields no default. This observation relies on the (realistic) assumption that breaking contracts entails costs but paying less because that is what a contract specifies doesn't.

The optimal  $\Delta d$  will depend on  $k$ ,  $d$  and  $s$ . As in equilibrium there is no default,  $V^h = V_{pay}^h$  and  $V^l = V_{pay}^l$ . However, the optimal  $\Delta d$  depends on whether the borrowing constraint is binding or not, that is, whether the country would borrow more in the absence of commitment problems. If the country is not constrained, the derivatives of the value functions with respect to debt have to be the same in both cases:

**Proposition 2** *If the borrowing constraint is not binding,  $\Delta d$  is chosen to make*

$$\frac{\partial V_{pay}^h(k', d^h)}{\partial d} = \frac{\partial V_{pay}^l(k', d^l)}{\partial d}$$

*Proof: see appendix.*

If the country is constrained, the optimal contract is different. Define  $mpk = Af'(k) - \delta$  and  $r^{*i} = 1/q^{*i} - 1$ .

**Proposition 3** *If the borrowing constraint is binding, so that  $mpk > r^{*i}$ , and  $|q^{*h} - q^{*l}|$  is small enough, then  $\Delta d$  is chosen to make  $V_{pay}^h(k', d^h) = V_{pay}^l(k', d^l)$ .*

Proof:

Suppose  $s = h$  and  $k', d', \Delta d$  are such that  $V_{pay}^h(k', d^h) > V_{pay}^l(k', d^l)$ . If the borrowing constraint is binding, then  $V_{pay}^l(k', d^l) = V_{def}(k')$ . By increasing  $\Delta d$  by  $dD$ ,  $V_{pay}^l$  increases and  $V_{pay}^h$  decreases. Now, the borrowing constraint is not binding anymore, so we can increase  $k'$  by some  $dk'$  and  $d'$  by  $dk'/q^{*i}$  while still respecting the borrowing constraint.

Utility at the present period hasn't changed.  $V^l(k', d^l)$  changes by

$$u'(c_L) \cdot [dk' \cdot (mpk - r^{*i}) + (1 - \psi) \cdot dD]$$

and  $V^h(k', d^h)$  changes by

$$u'(c_H) \cdot [dk' \cdot (mpk - r^{*i}) - \psi \cdot dD]$$

where  $c_L$  and  $c_H$  are consumption next period in the low and high state, respectively. So the change in  $V^h(k, d)$  equals:

$$\beta \cdot \{[\psi u'(c_L) + (1 - \psi) u'(c_H)] dk' (mpk - r^{*i}) + \psi(1 - \psi) dD [u'(c_L) - u'(c_H)]\}$$

Now, if  $|q^{*h} - q^{*l}|$  is small enough,  $(u'(c_L) - u'(c_H))$  is small and the change in  $V^h(k, d)$  is positive. A similar expression can be derived if  $V^l > V^h$  (see appendix). If  $s = l$ , similar expressions can be derived (see appendix).  $\square$

The optimal contract transfers resources from the higher to the lower state so that the borrowing constraint cannot be relaxed anymore by transferring debt across states. The benefits of the optimal contract are not related to risk-sharing: first-order benefits arise because relaxing the borrowing constraints enables the country to borrow more and, as its marginal productivity of capital is bigger than  $r^*$ , world output is higher.

## 5.2 The value of $\Delta d$

Let's consider that  $dq^* = q^{*h} - q^{*l}$  is sufficiently small and work with linear approximations.

For analytical convenience, let's consider a different process for  $q^*$ : at time  $t = 0$ ,  $q_0^* = q^{\xi}$ . From time  $t = 1$  on,  $q^*$  may return to  $\bar{q}$  at every period with a constant probability and, after it returns, we are in a deterministic world. So, for  $t > 0$ :

- if  $q_{t-1}^* = q^\xi$ ,  $\Pr(q_t^* = q^\xi) = \xi$  and  $\Pr(q_t^* = \bar{q}) = 1 - \xi$ ;
- if  $q_{t-1}^* = \bar{q}$ ,  $q_t^* = \bar{q}$ .

We will call this the  $\xi$ -process, as opposed to the  $\psi$ -process that we described above.

The value function at  $(k, d)$  if  $q_t = q^\xi$  is:

$$V^\xi(k, d, q^\xi) = u \left( Af(k) + (1 - \delta)k - k' - d + q^\xi d' \right) + \beta \left[ (1 - \xi)V^{\det}(k', d') + \xi V^\xi(k', d', q^\xi) \right] \quad (1)$$

where  $V^{\det}$  is the value function at  $(k, d)$  in the model with no uncertainty.

We want to relate the value functions in the cases of the  $\xi$ -process and the  $\psi$ -process. Suppose that we start at  $q^* = q^\xi = q^{*h}$  and define  $\bar{q} = (q^{*h} + q^{*l})/2$ . As a first order approximation, the risk of returning to  $\bar{q}$  with probability  $2\psi$  and staying there forever ( $\xi$ -process) and the risk of switching to  $q^{*l}$  with probability  $\psi$  and following the  $\psi$ -process from then on have exactly the same impact on the value function. The following lemma formalizes this statement.

**Lemma 4** Define  $\bar{q} = (q^{*h} + q^{*l})/2$  and denote by  $V^\psi(k, d, q^{*h})$  the value function at  $(k, d)$  and state high in the case of the  $\psi$ -process. Then  $V^\psi(k, d, q^{*h}) = V^\xi(k, d, q^{*h})$  if  $\xi = 1 - 2\psi$ .

*Proof: see appendix.*

Now, if  $q^*$  follows the  $\xi$ -process, let's compare the following two cases: (1)  $q^* = q^\xi$  and debt is  $d_0^\xi$  and (2)  $q^* = \bar{q}$  and debt is  $d_0$ . Which values of  $d_0^\xi$  and  $d_0$  make the country indifferent between both cases? By taking a linear approximation of  $V^\xi(k, d^\xi, q_0^\xi)$  around  $V^{\det}(k, d_0)$  and doing algebra, we get the result in the following lemma:

**Lemma 5**  $V^\xi(k, d^\xi, q_0^\xi) = V^{\det}(k, d_0)$  implies:

$$\begin{aligned} & u'(c_0) \left( d_0^\xi - d_0 \right) \\ &= \sum_{t=0}^{\infty} (\beta\xi)^t u'(c_t) (q^\xi - \bar{q}) d_{t+1} \\ & \quad + \sum_{t=0}^{\infty} (\beta\xi)^t \left[ u'(c_t)\bar{q} + \beta \frac{\partial V(k_{t+1}, d_{t+1})}{\partial d} \right] (d_{t+1}^\xi - d_{t+1}) \end{aligned}$$

where  $d_{t+1}^\xi$  is debt contracted at time  $t$  if  $q_t^* = q^\xi$  and  $d_{t+1}$  is debt contracted at time  $t$  if  $q_t^* = \bar{q}$ .

*Proof: see appendix.*

For expositional sake, suppose that  $q_0^\xi > \bar{q}$ . The first line in the above expression shows the utility cost of having a higher debt ( $d_0^\xi$  instead of  $d_0$ ). The second line shows the utility benefit of borrowing at a lower rate ( $q^\xi$  instead of  $\bar{q}$ ), and takes into account the probability of borrowing at a cheaper rate in future periods (so, depends positively on  $\xi$ ). The third term is the benefit of being able to borrow more due to the lower interest rates. If the borrowing constraint is binding, then

$$u'(c_t)\bar{q} > -\beta \frac{\partial V(k_{t+1}, d_{t+1})}{\partial d}$$

which means that the benefit of borrowing an extra unit this period is bigger than the cost of having an extra unit of debt next period. The lower is the level of capital, the higher is the marginal productivity of capital, the larger is the benefit of borrowing an extra unit. This benefit is decreasing in the level of capital.

We can write:

$$\frac{d_0^\xi - d_0}{d_0} > \sum_{t=0}^{\infty} (\beta\xi)^t \frac{u'(c_t)}{u'(c_0)} (q^\xi - \bar{q}) \frac{d_{t+1}}{d_0}$$

If we are close enough to the steady state, the inequality gets closer to an equality (borrowing constraint is less and less binding), and  $c_t$  and  $d_t$  are close to constant. In the limit:

$$\frac{d_0^\xi - d_0}{d_0} = \frac{q^\xi - \bar{q}}{1 - \beta\xi}$$

And that leads to the following proposition:

**Proposition 6** *Consider a deterministic steady state, around  $\bar{k}$ ,  $\bar{d}$  and  $\bar{q}$ , such that  $q^{*h}$  and  $q^{*l}$  are close to  $\bar{q}$ . Then, a linear approximation around the steady state implies that  $V(\bar{k}, d^h, q^{*h}) = V(\bar{k}, d^l, q^{*l})$  when:*

$$\frac{d^h - d^l}{\bar{d}} = \frac{q^{*h} - q^{*l}}{1 - \beta(1 - 2\psi)}$$

where  $\bar{d} = (d^h + d^l) \div 2$ .

*Proof:* As shown above and using lemma 4,  $V(\bar{k}, d^h, q^{*h}) = V(\bar{k}, \bar{d}, \bar{q}) \Rightarrow V^\xi(\bar{k}, d^h, q^{*h}) = V(\bar{k}, \bar{d}, \bar{q}) \Rightarrow (d^h - \bar{d}) / \bar{d} = (q^h - \bar{q}) / [1 - \beta(1 - 2\psi)]$ .

Analogously,  $V(\bar{k}, d^l, q^{*l}) = V(\bar{k}, \bar{d}, \bar{q}) \Rightarrow (d^l - \bar{d}) / \bar{d} = (q^l - \bar{q}) / [1 - \beta(1 - 2\psi)]$ . Using both equations, we get the claim.

Thus  $\Delta d$  depends basically on:

1. the magnitude of interest rate fluctuations ( $q^{*h} - q^{*l}$ );
2. the persistence of interest rate ( $\psi$ );

3. the current level of capital — and its marginal productivity.

Closer to the steady state, the key variables for determining  $\Delta d$  are the size of interest rate fluctuations ( $q^{*h} - q^{*l}$ ) and the persistence of interest rate ( $\psi$ ). Higher persistence implies higher difference between  $d^h$  and  $d^l$ . In the i.i.d. case,  $\psi = 0.5$ , and  $d^h - d^l = d^l (q^{*h} - q^{*l})$ , that is, the debt at the low state has to decrease to exactly compensate the smaller borrowing. In the other extreme,  $\psi \rightarrow 0$ ,  $d^h - d^l = d^l (q^{*h} - q^{*l}) / (1 - \beta)$ , that is, the debt reduction in the low state must be much bigger to compensate the expected future loss brought by the fall in  $q$ .

The lower is the level of capital, the bigger are the marginal productivity of capital and the difference between  $u'(c_t)q^1$  and  $-\beta \frac{\partial V(k_{t+1}, d_{t+1})}{\partial d}$ , which contributes to increase  $\Delta d$ : a switch to the low state hurts more for preventing the country to borrow and grow (which tends to increase  $\Delta d$ ). A lower level of capital also implies a lower consumption and, therefore, higher marginal utilities, so the present is more important and the higher costs of borrowing in the future are less relevant (which tends to reduce  $\Delta d$ ). Last, a higher ratio between future and present debt increases a bit the importance of future costs of borrowing (which tends to increase  $\Delta d$ ). Thus the overall effect cannot be deduced by the formula. The numerical results show that  $\Delta d$  is decreasing in  $k$ , the effect of the borrowing constraint seem to predominate.

$\Delta d$  is not importantly affected by (i) the length of debt contracts and (ii) the output cost ( $\gamma$ ). If  $\beta = (q^1 + q^2)/2$ , it makes very little difference the length of the contract for the optimal  $\Delta d$ . For example, if interest rates may be 0% or 4% a year and  $\beta = (1.02)^{-1}$ , one-year contracts and  $\psi = 0.1$  yields  $(d^h - d^l) / d = 0.1783$  and five-year contracts and  $\psi = 0.5$  yields  $(d^h - d^l) / d = 0.1781$ . The output cost  $\gamma$  has no first order effects on  $\Delta d$ , it is just important to determine the level of debt.

The analysis has focused on the binary case because it is easier to obtain and understand the analytical results and to get numerical results, but the same insights apply if we consider more general processes. The next proposition considers the case of an auto-regressive process for  $q^*$ .

**Proposition 7** *Suppose that  $q^*$  follows an AR(1) process:*

$$q_{t+1}^* - \bar{q} = \zeta(q_t^* - \bar{q}) + \varepsilon_{t+1}$$

*and  $\text{Var}(\varepsilon_t)$  is arbitrarily small. If the economy is close to its steady state ( $k \simeq k^*$ ),  $V(k, d^1, q^{*1}) = V(k, d^2, q^{*2})$  when:*

$$\frac{d^1 - d^2}{\bar{d}} = \frac{q^{*1} - q^{*2}}{1 - \alpha\zeta}$$

where  $\bar{d}$  is the level of debt in the deterministic model when  $q = \bar{q}$ .

*Proof: see appendix.*

### 5.3 The Latin American debt crisis of the 80's

External shocks were important factors in the Latin American debt crisis of the 80's. As pointed by Diaz-Alejandro (1984), countries with very different policies (some very laissez-faire, some very interventionist), very distinct economies (some oil exporters, other oil importers) ended up in the same crisis situation in the beginning of the 80's, facing problems that in 1979 would have been considered very unlikely. One key external shock is the increase in US real interest rates, shown at figure 3 (from Dotsey et al, 2003). While in the 70's, real interest rates were around 0%, in the 80's they were around 4% (and higher in the first half of the decade).<sup>7</sup>

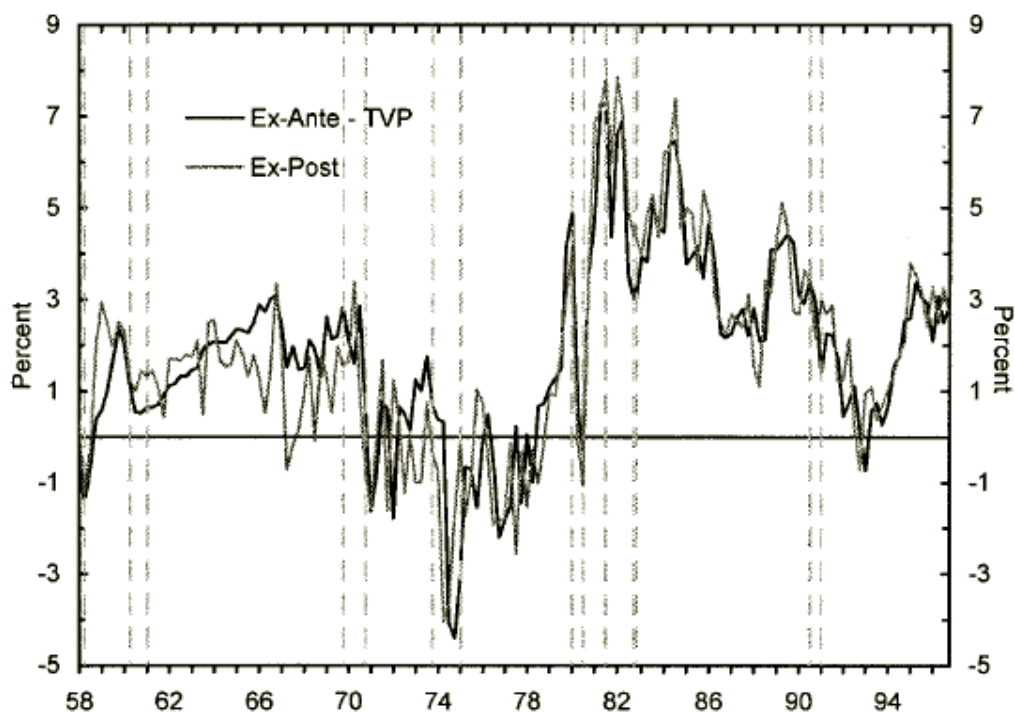


Figure 3: US real interest rates

In the beginning of the 80's, the prices of Latin American bonds in secondary markets went down, capital flows to those economies dried or reverted and the fast process

<sup>7</sup>There are other cases in which interest rate increases in the US contributed to crises in other countries. The hike in US interest rates in 1994, for example, is often mentioned as one of the factors that let Mexico very close to defaulting in December/1994.

of economic growth of the 70's stopped. After countless IMF missions, several debt rescheduling and interest arrears and some attempts of debt renegotiation (including the Baker Plan), came the Brady agreements. In the period between 1989 and 1994, most of the main Latin American countries got some debt relief. Table 1 shows the debt relief following the Brady Plan agreements as a percentage of the outstanding long term debt in the main Latin American countries. Figures are mostly around 30% and vary between 20% and 31%.<sup>8</sup>

Table 1: Debt relief – Brady plan agreements

Venezuela	20%
Brazil	28%
Argentina	29%
Mexico	30%
Uruguay	31%

In the model, the optimal contract prescribes automatic debt relief in order to compensate for unexpected increases in interest rates. In reality, debt relief came ten years later, and what happened in between also had some influence in the final agreement. Nevertheless, it is worth comparing the debt relief prescribed by the model and the one observed in reality, despite the 10-year delay, because the Brady agreements are in fact mostly the solution of a problem that erupted in the beginning to the 80's.

### 5.3.1 Debt relief according to the model

The key implication of the optimal contract is that debt relief compensates the country for unexpected increases in interest rates. If those countries' borrowing constraints were binding in 1979, the high interest rates of the beginning of the 80's brought  $d$  above its incentive compatible level. The question is: how much above?

Consider the model with 2 possible states and suppose that world interest rates may be either 0% or 4% a year and that each state lasts for an average of 10 years:  $q^{*h} = 1.00$ ,  $q^{*l} = 1.04^{-1}$ ,  $\beta = 1.02^{-1}$  and  $\psi = 0.10$ . When  $q^*$  switches from high to low, what should be the optimal repayment? The analytical solution (linear approximation) yields  $\Delta d = 0.178$ . That implies a spread over treasury of 1.8% when the state is high and

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<sup>8</sup>As noted by Cline (1995), the initial approach for dealing with the problem of debt overhang was aimed not only at reducing debt but also at providing new loans, but “for practical purposes the Brady Plan has been all forgiveness and no new money” (Cline, 1995, page 236). Indeed, according to the model, if the amount of debt exceeds its incentive compatible level, new money won't come in.

debt relief of 16% when the state switches to low and interest rates jump from 0% to 4%. Using the auto-regressive process and assuming a half-life of 3 years for the interest rate increase, the AR-1 coefficient,  $\zeta$ , would be 0.79. A jump in real interest rates from 1% a year to 6% a year would then imply an even bigger debt relief:  $\Delta d = 22.5\%$ .

According to the model, the decrease in the level of debt in response to an interest rate increase of such magnitude exceeds half of the observed debt relief of the Brady agreements. As the interest rate hike is considered to be an important factor on the crisis but not the only one and this computation considers only its direct effects on the debt (and not the indirect effects through less imports, etc), I take that as a positive sign of adequacy of the framework presented in this paper and of the importance of movements in US interest rates for the Latin American debt crisis of the 80's.

Those results come from the analytical approximations. How accurate are they?

#### 5.4 Numerical results

The two key take-home points of the numerical exercises presented below are:

1. The formula presented at proposition 6 is a good approximation for  $\Delta d$  as long as  $mpk > r^{*l}$ , that is, as long as the country is constrained regardless of the state of the economy.
2. With debt contracts that imply  $V^h(k, d^h) = V^l(k, d^l)$  at every state, even if the borrowing constraint is not binding, welfare may be smaller than with uncontingent bonds at some states.

I obtain the values of  $\Delta d$  that make  $V^h(k, d^h) = V^l(k, d^l)$  at every state. To do so, starting from some initial guess for  $\Delta d$  (zero everywhere), I obtain the optimal decisions on debt and capital, as well as the value of  $q$  at every state. Then, I calculate (numerically) the derivative of the value functions with respect to debt and obtain an estimate of  $\Delta d$  that would make  $V^h(k, d^h) = V^l(k, d^l)$ . The process is iterated until convergence.

Calibration: one period correspond to one year,  $\alpha = 0.36$ ,  $\beta = 1.02^{-1}$ ,  $\gamma = 0.01$ ,  $\sigma = 3$  and  $\delta = 0.10$ .  $A = 1$ , and  $q^*$  fluctuates around  $\beta$ :  $q^{*l} = 1.04^{-1}$  and  $q^{*h} = 1.02^{-1}$ . As before,  $k' - k$  is constrained to lie in some interval — adjustment costs for capital are 0 in a certain range and infinity beyond that.

Figure 4 shows  $\Delta d$  as a function of the marginal productivity of capital ( $mpk = \alpha Ak^{\alpha-1} - \delta$ ) if the borrowing constraint is binding and state is high in two situations:

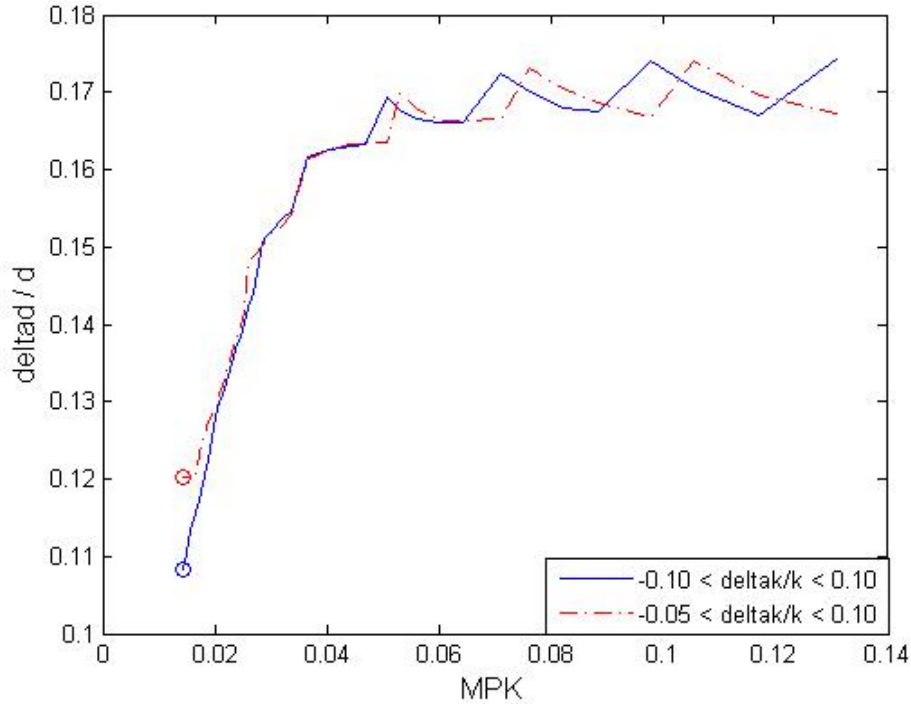


Figure 4:

(a)  $k' - k \in (-0.10k, 0.10k)$  and (b)  $k' - k \in (-0.05k, 0.10k)$ . The main results are the following:

- For  $mpk > 4\%$  (which corresponds to  $r^*$  at the low state), the linear approximation works well:  $\Delta d$  is around 0.17 and slowly increasing in  $mpk$ . The possibility of borrowing a bit more in the high state is worth more for countries with high  $mpk$ , but the difference is small.
- For  $mpk$  below  $r^{*l} = 4\%$ ,  $\Delta d$  is considerably smaller. For lower values of  $mpk$ , when the state shifts to low, interest rates are higher than its  $mpk$ , so the country sells its capital and makes profits by buying high-interest-rate foreign bonds. This sounds not too realistic, adjustment costs for capital would prevent such rapid capital reallocation and, indeed, the optimal  $\Delta d$  for lower values of  $mpk$  are sensitive to the assumptions on adjustment costs. Figure 4 shows that when the level capital is high enough so that the country decides not to change while the state keeps being high ( $mpk$  below 2% a year),  $\Delta d/d$  is below 0.11 with adjustment costs given by (a) and around 0.12 in case (b).

- The welfare impact of such contracts may be negative. For a country with high levels of  $k$ , issuing debt contracts that imply  $V^l(k', d') = V^h(k', d')$  at all states is worse than borrowing through uncontingent bonds. The optimal contracts in the unconstrained case — that imply  $\frac{\partial V_{pay}^h(k', d^h)}{\partial d} = \frac{\partial V_{pay}^l(k', d^l)}{\partial d}$  — may prescribe very different values of  $\Delta d$ .
- The welfare of a country with no debt,  $k = 2$  (that is, one third of  $k^*$ ) and debt contracts designed to make  $V^h(k', d^h) = V^h(k', d^l)$  is equivalent to the welfare of a country with no debt, access to uncontingent bonds and  $k = 2.0676$  (that is, 3.4% more capital). This difference can only grow if we choose the appropriate debt contracts when the country is not constrained.
- The model with 5-year intervals yields very similar quantitative results (and the same qualitative results), confirming that the time-to-maturity of debt is not an important variable in this model.

Coming back to the comparison with the Latin American debt crisis of the 80's: the implications of the model depend on the marginal productivity of capital in those countries. If it was not lower than  $r^{*l}$ , the numbers from the analytical results are very good approximations and the interest rate hike of the beginning of the 80's would imply a debt relief of more than half of the observed haircut obtained through the Brady agreements. Lower marginal productivity of capital (combined with low adjustment costs for capital) reduce the debt relief prescribed by the model.

## 6 Practical issues on contingent debt

Following the Latin American debt crisis of the 80's, many suggestions for indexing debt to some real economic variables have been made. Some popular variables are GDP (Athanasoulis and Shiller (2001) and Borenzstein and Mauro (2004) are some recent references) and commodities prices (see, e.g., Kletzer et al (1992)). Much of the discussion was about moral hazard issues (see, e.g., Krugman, 1988): indexing debt to GDP would reduce incentives for governments to make effort.

Many of the problems related to GDP-indexed bonds don't apply to contracts contingent on world interest rates:

- there is no moral hazard (see, e.g., Krugman (1988));

- no major measurement problems, danger of misreporting, data revisions, lag in data announcements, we only need to estimate expected inflation in the relevant developed countries;
- while countries' GDP's are positively correlated,  $r^*$  and  $y^*$  are negatively correlated. As shown by Neumeyer and Perri (2005), interest rates and output are positively correlated in developed countries but negatively correlated in emerging markets.

A final question would be: if it is that good, why hasn't the world done it yet? As argued by Borensztein and Mauro (2004), there may be many obstacles to innovations in sovereign debt, for example: policy makers are usually considered to have short horizons and preferences different than the agents; there may be fixed costs to introduce a different kind of debt — and so a free rider problem and, perhaps, an equilibrium in which noone pays the fixed cost.

Even if contingencies are implicitly considered, given the very high costs of debt renegotiations and defaults, it would be desirable to make such connection explicit in order to save the heavy retaliation and bargaining costs — the recent dramatic fall in Argentinean GDP after its default is just one example of such costs. The fact that such heavy bargaining costs are paid not only by creditors and debtors but also by international organizations (e.g., the IMF) reduce incentives for creditors and debtors to write such contracts.

## 7 Concluding Remarks

Lucas (2003) makes a strong case for macroeconomists to focus on growth issues rather than business cycles because welfare impacts of a little bit of extra growth are much bigger than the effects of smoothing fluctuations. This paper studies debt and default in the context of a growth model and analyzes how differences between a country's marginal productivity of capital and the world interest rates and fluctuations in such differences lead to debt and default. In this context, contingent debt contracts (implicit or not) bring first order gains by allowing more capital to flow to where its marginal productivity is higher.

Abstracting from debt renegotiation allows for great tractability in the model, but at a cost: the bargaining issues are overlooked in this paper. Still, I expect the results to survive in an environment in which bargaining is explicitly considered.

In the model, frequent debt relief (or partial default) is part of the optimal equilibrium and, differently from what is suggested by Reinhart and Rogoff (2004), is better than a

situation with very little foreign lending. However, the bargaining costs related to debt defaults have been very high and making debt contingent would help. Some recent policy prescriptions focus on such bargaining costs — the IMF’s sovereign debt restructuring mechanisms is the main example (see, e.g., Krueger (2002)). At least in the case of the Latin American debt crisis of the 80’s, having debt explicit contingent to world interest rates would have avoided most of the problem.

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