A Cellular Automata Model of the General Rate of Profit*

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Abstract

We present a simulation exercise of classical free competition in which individual capitals attracted by prospective rates of profit move across industries in their Moore neighborhoods. Capitals, in general, never settle down to a fully equalized general rate of profit position and the most common characteristic of the series of cross sectional average rate of profit is the never ending gravitation around the average rate of profit determined by the number of capitals in the lattice-economy. The statistical properties that emerge from the interaction of our agents resembles stable distributions chracterized by skewness and heavy tails.

JEL Classification: C15, D58, E11

Keywords: Classical Free competition, General Rate of Profit, Agent-based models, Stable Distributions

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List of Figures

Introduction

In this essay we present an agent-based model in which profit-maximizing capitals move across industries according to perceived differentials in prospective rates of profit. Capitals follow a simple algorithm that identifies the set of industries with maximum expected profit rates in a given virtual neighborhood of a lattice economy and then move capitals to a site selected randomly from this set of maxima. The likelihood that a capital will move to sites with higher rates of return depend on a probability function. 'Probability to move' is a Probit function positively related to the number of neighbors with a maximum rate of profit and negatively related to the size of the individual capitals. Hence, we have a stochastic model that assumes deterministic form as the parameters of the Probit function approach extreme values. Notwithstanding the fact that our model does not incorporate accumulation of capital, an important part of our simulations is based of the variation of the total amount of capitals in the economy, as well as in size of agents.

Although the dynamics of the model are created mainly by differences in expected rates of profit across industries, agents' bounded rationality will play a fundamental role in the determination of the trajectories of the cross-sectional average rate of profit during the simulation. The degree of information about differential profit rates, represented by the size of the neighborhoods, as well as discrepancies between perceived and realized rates of return will substantially modulate the basic structure of the system and create non-trivial paths in the pre-asymptotic phase of the process of equalization of rates of profit.

Such representation of the movement of profit maximizing firms in a fictitious economy extends to 2500 sectors Adam Smith's (1937, p.87) exposition of the formation of a general rate of profit under conditions of free competition. As it is widely known, Smith's ingenuous allegory of the equalization of rates of profit between a Beaver and a Deer hunting activities - and the consequent determination of a vector of so-called natural prices corresponding to long-period conditions of production - served as basis for Ricardo's famous corn model and consequently for contemporaneous attempts to correct and reinstate classical price theory in modern terms (Sraffa, 1960; Schwartz, 1961).

One of the interpretations of the legacy of classical political economy was the idea that the set of natural prices corresponding to the general rate of profit represented a long run static equilibrium much similar to those found in the walrasian perfect competition tradition. This somewhat dubious version of classical notion of free competition seems to draw its conclusions from a particular perception of the Smithian idea of a emergent order established by independent producers and consumers without any explicit coordination beyond the everyday higgling mechanism of market exchanges.

Much controversy has been generated among contemporaneous economists of non-neoclassical inspiration on the meaning, convenience and particularly on the empirical robustness of the assumption in their models of a single, equalized rate of profit for the reconstruction of a comprehensive classical-inspired alternative to marginalism. Farjoun (1983), Shaikh (1980, 2000) have argued that the strength of classical political economy - and its irreconcilability with the Walrasian weltanschauung - lies in its inherently dynamic character. For these authors, the main strength of the notion of classical free competition is rather located in the accounts of the rich disequilibrium processes - the never ending turbulent adjustments of the constellation of market prices towards prices of production - than in the focus on the static equilibrium part of the Deer-Beaver story.

In this work we are interested in two specific dimensions of the aforementioned problem. First, to observe under what conditions capitals settle down to a fixed-point equilibrium, and if it corresponds to a single, equalized rate of profit across sectors of the economy. Second, to investigate what are the statistical processes that characterize the adjustment process towards long-run atractors. We strictly follow the level of abstraction of the examples presented by Smith in the part I or the Wealth of Nations and we believe that our simulations may be able to shed some substantial light on the meaning of the general rate of profit in the tradition of Classical political economy.

Our main results could be summarized as follows:

Capitals in most of our simulations do not settle to a fixed average rate of profit. In most cases, quasi-steady state is attained, through the establishment of a regular frequency of variation of the average profit rate around a fixed point. Hence, if we were to talk about convergence of profit rates to a general rate of profit, this is mostly observed in the convergence of its variance. This seems to give support to the idea of the general rate of profit as an long-run average of a turbulent gravitation mechanism, rather than a verifiable static equilibrium.

A fixed point equilibrium is attained only at the very restrict case when total amount of homogeneous capitals are equal (or multiple) of the number of industries (nodes) in the lattice. This case resembles Edgeworth's argument that the existence of a general equilibrium is only attained through the assumption of divisible goods.

As the number of capitals increase, the cross-sectional average rate of profit series tend to present a more well defined convergence to a mean intercalated by erratic patterns of chaotic fashion.

Finally, the statistical processes that emerge from our agent's interaction is mostly non-stationary with characteristics of stable distributions where leptokurtic, skewed distribution are combined with long term autocorrelation.

This paper is organized as follows. In section 1 we present a brief survey of the debate on classical competition. In section 2 we present the model. Section 3 presents the simulation results and Conclusion sums up the main results and presents suggestions for further research.

1. Smithian Competition

Adam Smith's competition theory is presented in its final form in the Wealth of Nations, and represents the most important elaboration of one of the central categories of political economy - the notion of *free competition.* In his work, generalized competition was the crucial element that, in one hand, provided the necessary cohesion for a system that, notwithstanding being constituted by a multitude of self-interested independent and uncoordinated producers, still (but not always) produced outcomes that were of public benefit. On the other hand, competition also created quite distinct and determinate patterns of development of this system which were particular to the new form of society he named "commercial civilisation".

Smith believed that it was possible to understand the mechanism behind those patterns by systematic application of analogy from some other science (or art) , in which fruitful discoveries had been made. [1] The discovery of the chain of causal relations in a system implied the identification of its so-called laws of motion and, by consequence, the deviations from those laws. His comments on Aristotle echoed his perception of this general principle of scientific method:

Some of these sensible qualities, therefore, we regarded as essential, or such as showed, by their presence or absence, the presence or absence of that essential form from which they necessarily flowed: Others were accidental, or such whose presence or absence had no such necessary consequences. The first of these two

[1] Although Smith seems to have found in the physics of Newton a major source of inspiration

for his work of connecting the principles of social system, D. Redman (1993) argues convincigly that the similarieties with newtonianism are more retoric than methododologic as Smith did not borrow a mechanistic view of the works of the society. Thomson (1965), also sees the influence of Hume's philosophical skepticism on Smith's views on (the limits of) theory, seemed as a innerently methaphysical elaboration:

Even we, while we have been endeavouring to represent all philosophical systems as mere inventions of the imagination, to connect together the otherwise disjointed and discordant phaenomena of nature, have insensibly been drawn in, to make use of language expressing the connecting principles of this one, as if they were the real chains which Nature makes use of to bind together her several operations. Can we wonder then, that it (Newtons System) should have gained the general and complete approbation of mankind, and that it should now be considered, not as an attempt to connect in the imagination the phaenomena of the Heavens, but as the greatest discovery that ever was made by man, the discovery of an immense chain of the most important and sublime truths, all closely connected together, by one capital fact, of the reality of which we have daily experience.(Smith, 1905, pp. 189-90.)

sorts of qualities was called Properties; the second, Accidents (ibid).

Smith's account of competition asserts that under free competition differential in rates of profits among industries tend to be equalized by flows of investment that migrate from areas with lower returns to those who present more attractive ones. The inflow of capitals to a high-return industry, given the *effectual demand*, ultimately leads to an excess in the supply of goods and consequently causes a fall in market prices that will bring down the rates of profit. This phenomenon brings about an outflow of capital to another industry with better profitability and the process repeats all over again. The main results of this simple model are a tendency for the rate of profit to equalize across different industries, the formation of a vector of prices corresponding to this equalized rate and a gravitation of market prices to or around this set of equilibrium prices. Smith adopted an usual terminology of his time and called these prices of equilibrium *natural prices*.

As wages and technology are exogenously given in his model, the set of natural prices are related to a set of outputs. These quantities are the shares of each sector on total output and correspond to the (also given) effectual demand facing each industry.

Although we have here the combination of a supply and demand mechanism and a tendency towards a form of equilibrium, classical free competition is still very different from perfect competition stories. Classicals do not see firms as price-takers and their entrance in a new market has well determined negative effects on the market price as supply is increased. Although classicals assume a negative relation between quantities demanded and market prices, there is no assumption of a fully determined demand curve and therefore no direct linkage between market price relations and value and distribution theories.

However, the most contrasting difference between classical and neoclassical approaches is the emphasis by the former on the process of adjustment towards equilibrium. It is the focus on positions of disequilibrium that constitutes the main strength of Smith, Ricardo, and especially Marx's theory. This is neither the case in the Marshallian partial equilibrium tradition, where the center of analysis starts to shift to market clearing positions[2], nor in the Walrasian general equilibrium models, where *ex ante* market clearing process is assumed and no transaction takes place before 'the right' set of prices is established through *tâtonnement.*

The traverse towards long-run equilibrium in the classicals assumes that, in real economies, non-market clearing transactions in the market for commodities *must* occur so that adjustments of inventories and stock of capital to the necessities

[2] Marshall's notion of short-run prices of temporary equilibrium is pretty similar to Smith's market prices. However, an important conceptual difference lies in the *ad hoc* assumption of the former as a market clearing price.

of effectual can take place. It is only through this progressive form of turbulent regulation that enterprises are 'informed' of situations of disequilibrium between supply and demand and can accordingly adjust their productive capacity towards full adjustment. If market equilibrium ever occurs, it can only be verified *ex post*. Hence, the idea of analyzing equilibrium relations without an account of disequilibrium processes is largely foreign to the classical analysis.

Notwithstanding this fact, controversy was created among modern economists on the proper treatment of long run positions for the construction of a contemporaneous analysis of capitalism based on the classical approach. Although the debate was more evident in issues related to the notorious 'transformation problem ', it also encompassed independent dimensions of price and competition theory. In what concerns the latter, the question of the possibility to demonstrate that sequential price models with classical properties could converge to equilibrium prices of production occupied great part of the debate among post-classical economists.

Smith indicated many times in his writings that the he saw the convergence to prices of production to take place in the very long run, therefore seeming to suggest that the properties of the never-ending gravitation to or around those prices may be of equal or greater importance for our understanding of capitalism:

But though the market price of every particular commodity is in this manner *continually gravitating, if one may say so, towards the natural price, yet sometimes particular accidents, sometimes natural causes, and sometimes particular regulations of police, may, in many commodities, keep up the market price, for a long time together, a good deal above the natural price.(Smith, 1937, p. 12)*

In spite of this evidence, the debate among post-classical authors on the relation between fully adjusted positions and gravitation properties seems to require at least more clarification.

Dumenil and Levy (1987) suggested that convergenge of market prices to prices of production assumes the form of a quite determined form of attractor:

> *The situation towards which the economy ultimately tends is a 'fixed point', i.e., an 'equilibrium' of the dynamic process. This is especially evident in Marx's analysis, where the word 'equilibrium' is repeatedly mentioned in this respect.(p. 24)*

And although they stress the disequilibrium properties of the classical model, this seems to be mostly seem as caused by changes in the exogenous parameters of the model or by random shocks[3]:

[3] Although the authors do admit *en passant* the possibility that the gravitation properties of the Classical model be caused by some endogenous imperfection in the convergence process factor.[p.151, fn.1]

> *The equilibrium position[...] is a target that is never actually reached. Instead, prices and outputs 'gravitate'[...] around this target, because of systematic forces of change (technical progress, demand changes, etc), or as a result of the impact of constant perturbation or persistent shocks to the economy.*(ibid, p. 25)

Farjoun and Machover (1983) take a different stand:

> *Our main aim is to show that this generally accepted belief rests on a fundamental theoretical misconception: there is no way in which the logic of the capitalist system - let alone its reality - can be encapsulated by a model in which the rates of profit accruing to all productively invested capitals are assumed to be equal (p. 14)*

Finally, Shaikh (2000) provides a balanced perspective on the same question:

> *The unplanned individual activities which characterize capitalist production are made socially coherent only by being forcibly articulated into a viable social division of labor, through some real process of oscillations, discrepancies, and errors around ever moving centers of gravity. It is one thing to study the properties of these centers of gravity, as the classicals do in their analysis of prices of production or of balanced reproduction. But it is quite another to assume these conditions ever exist as such, or that one may analyze the behavior of individual units beginning from some assumed state of equilibrium (as modern economists so often do).(p. 3)*

Considering this controversy, we believe that still today the analysis of the behavior of sectoral rates of profits in the process of emergence of a general rate of profit has been relatively unexplored. The characteristics of the path towards a rate of profit of equilibrium - if this happens to exist - can provide us with some significant insights on the meaning of fully adjusted positions in the works of the classicals and its relevance for the understanding of real economies.

2. The Model

The model is *played* on a $n - by - n$ lattice with C_{ij} cells corresponding to each (i, j) node. Our simulations assumed a $50 - by - 50$ lattice-economy in which each of the 2500 cells represented a different sector - in marxian terms, a sphere of production.

Capitals

The total value of capital K in each site of the lattice is the sum of individual capital $k^1, ..., k^n$ at node $[i, j]$

$$K_{(i,j)} = \sum_{l=1}^{n} k^l_{(i,j)} \qquad l = 1, ..., n \tag{2.1}$$

Individual capitals are heterogeneous, i.e., they can have different sizes. However, at our level of abstraction, we assume a very simplified productive structure in each sector and we do not allow for differences in capital compositions or technology. Therefore, for the purpose of simulation, our stylized sphere of production behaves as if it only contained one firm a series of firm with the same cost structure and therefore a single set of market prices[4]. The general rate of accumulation

[4] Thus, we are not investigating questions related to the determination of market-values and the role of regulating capitals in the determination of the general rate of profit.

of capital is equal to the depreciation rate and the total number of capitals in the lattice is assumed constant.

Rate of Profit

Finally, the rate of profit $r_{(i,j)}$ at each node (i,j) is as non-linear function of the total capital in the form

$$r_{(i,j)} = e^{-K_{(i,j)}} \tag{2.2}$$

Neighborhoods, Boundary Conditions and Limits to Rationality

The structure of our model assumes that each sphere of production faces a Moore neighborhood N^{Moore}- an array of surrounding cells in square form that will directly influence the value of our given cell at site (i_o, j_o) in a radius rad and is defined formally as:

$$N^{Moore}_{(i_o,j_o)} = \{(i,j) : |i - i_o| \leq rad, |j - j_o| \leq rad\} \tag{2.3}$$

The boundaries of the economy are periodic, meaning that each $4(n-1)$ cells at the borders of the lattice are 'wrapped' so that, for example, cell $C_{(1,100)}$ will face cell $C_{(1,1)}$ at the opposing border.

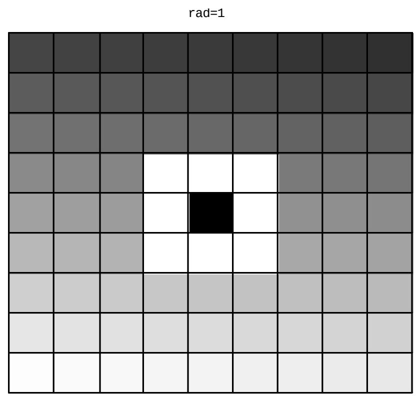

Figure 2.1: A Moore Neighborhood with radius=1

We are interested in comparing results in which capitals have perfect knowledge on profitability conditions in the entire economy with situations where this access to information is limited. In most of our results, our agents will have knowledge of the state of the economy and possibility of movement of capital restricted, in each round of the simulation, to their own (changing) Moore neighborhood . This does not mean that capitals cannot move freely around the lattice but that it takes time for them to get information about sectors of the economy with which they are not directly related. We assumed a moore neighborhood of radius equal to one.

Stochastic and Deterministic Rules

The basic algorithm for the evolution of our cellular automaton can be stated as

$$(r_{(i_o,j_o)} \notin S^{\max}_{(i_n,j_n)}) \Rightarrow \forall k^l_{(i_o,j_o)} \rightarrow k^l_{(i_n,j_n)} \qquad l = 1, ..., n. \tag{2.4}$$

Agents in a given node $[i_{o,}j_o]$ of the lattice will search for the maximum rate of profit in their moore neighborhood. In case the rate of profit in their own industry is smaller that those in the set of maximum rates of profit $S^{\max}$, they move their individual capitals k^l to a randomly selected node of $S^{\max}$ with probability $p = \Phi[x]$, where Φ is the operator that gives us the greatest integer smaller or equal to the Probit function χ defined as

$$\chi = [1 + (\frac{1}{1 + e^{-\kappa k + \alpha R}}) - \epsilon] \qquad 0 \leq \epsilon \leq 1 \tag{2.5}$$

where k is the size of each individual capital, R $(0 \leq R \leq 8)$ is the number of cells in the neighborhood of a given industry with a maximum rate of profit (i.e., the number of elements in the set $\mathrm{S}^{\max}$), κ and α are the respective parameters and ϵ is a random shock.

Basically, the function p could be interpreted as each capital's willingness to move, represented by the sensitivity to the amount of nodes in the neighborhood with higher rates of profits and to the size of a individual stock of capital. The rationale for such an assumption are many fold: capitals 'know' that the likelihood that other capitals will move to the same targeted cell with maximum rate of profit if there are only a few such cells in her neighborhood or that information on profitable industries is more scarce when less industries in the neighborhood have maximum rates of profit. In the case of the sluggishness in the response of firms with higher capital stock, represented by $(-\kappa k)$, could be seem as a barrier to exit as firms have more capital invested in a given industry (Semmler, 1984, p. 35).

Furthermore, P allow us to calibrate κ and α so that as these parameters grow, the stochastic model becomes deterministic.

This general specification allow us to investigate the different properties of both types of models and compare their results in a very convenient form as parameters are changed.

3. Results

Our model is therefore basically dependent on the given amount of capitals in the economy and on the 'vision' of agents, represented by the radius of the Moore neighborhood. As it was mentioned above, our models assumes stochastic char-

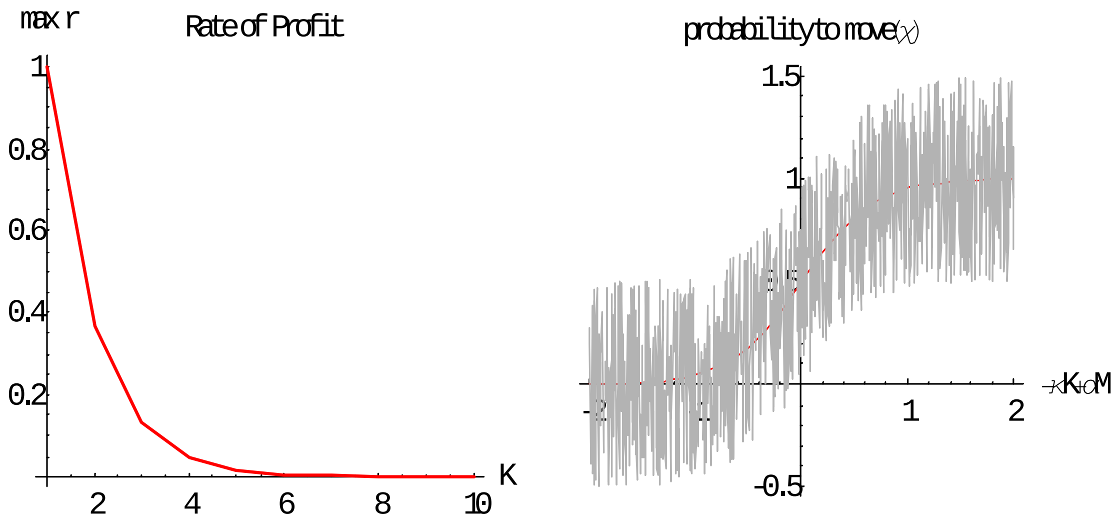

Figure 2.2: The Rate of profit is a natural exponential function of the stock of capital. Capitals move across the lattice according following a probit function with random shocks.

acteristics as the parameters of the probability function reach threshold values and this makes the movement of capitals also dependent on the size of individual capitals, the number of neighbors with higher rates of profits and their respective elasticities.

In this initial work we will limit our simulations to the case of homogenous individual capitals (therefore assuming a constant k), as negative results under this assumption holds *a fortiori* for the case of heterogenous capitals. Therefore, great part of our analysis will focus on changes in parameters α, κ, the total amount of capital K, the total number of nodes-industries N and on the maximum rate of profit neighboring nodes facing a given node R.

The presentation of our results will be divided in two general parts. First, we will investigate if, and under what conditions, capitals settle down to a general, equalized rate of profit. In the second part we will try to identify what specific statistical process characterizes the movement of the average rate of profit in its pre-asymptotic phases. Otherwise noticed, our conclusions will be based on the results of 500 recursions on a 50x50 lattice.

3.1. The general profit rate as a fixed point equilibrium?

3.1.1. Homogenous capitals with limited vision

Figures 3.1 to 3.11 present the results of simulations with variable amounts of total (homogenous) individual capitals. Our findings can be summarized as follows:

- $\alpha = 1$, $\kappa = -1$. **Trajectories a la Smith-Marx: Quasi-steady states**

With the parameters set to these values we have a totally deterministic model in which capital always move if nodes in the particular neighborhood present a higher rate of profit. Changes in the total amount of capital gave us the following broad patterns:

$K \leq N$

The main feature of the average rate of profit in cases in which total number of individual capitals is either smaller or equal to the number of industries in the lattice-economy (figs. 3.1 to 3.2) is a tendency for the average rate of profit series to wander around without a clearly determined trend or pattern in its movement. It resembles a highly erratic and jumpy behavior of a white-noise process, but we should not forget that these series were created from the interaction of agents following totally deterministic rules.

The same pattern is observed in the variance and standard deviation series, as well as in higher moments of the cross-sectional distribution. The cross-sectional skewness varied repeatedly from circa -1.1 to -0.7, without any trend. The kurtosis ranged from circa 1.8 to 2.4 hence demonstrating that the distribution of rates of profit is flat-topped, a result which is quite different from that one would expect if a tendency to an equalized rate of profit were to be found.

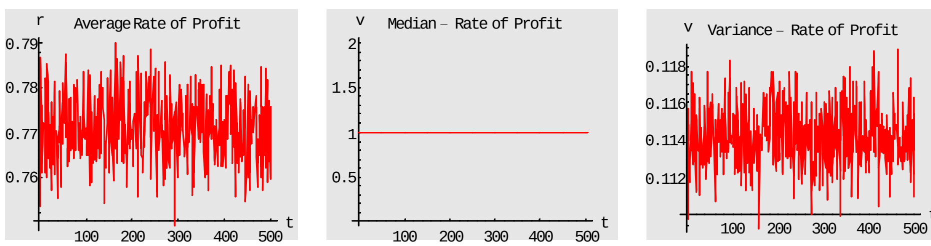

Figure 3.1: Rate of Profit - Descriptive Statistics: 1000 k

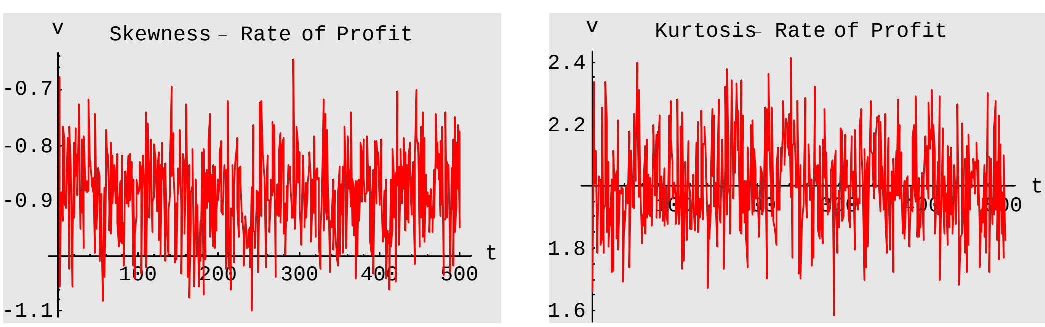

Figure 3.2: Rate of Profit - Higher moments of distribution: 1000 k

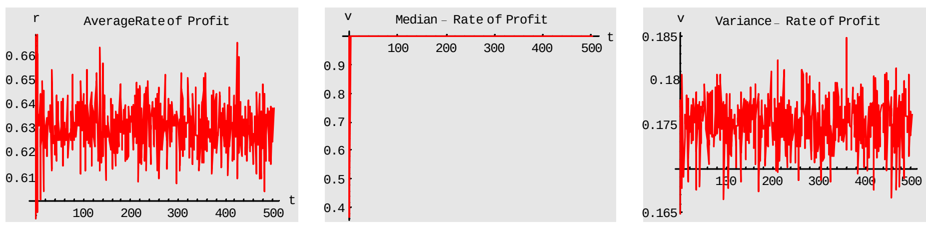

Figure 3.3: Rate of Profit - Descriptive Statistics: 2500 k

$K > N$

Elements of chaos: $K = 3N$

As total the total number of capitals in the lattice is increased to 7500 capitals, the previous (lack of) pattern turns into a more clearly discernible low-variance gravitation of the rate of profit around an imaginary mean. This is observable in figs. 3.4 to 3.6, where right after the recursions correct the initial and arbitrary placement of capitals, the series establishes its more characteristic movement, which is interrupted by sudden increases in its volatility, thus resembling the behavior of chaotic series. Notice, however, that the decrease in variance of the

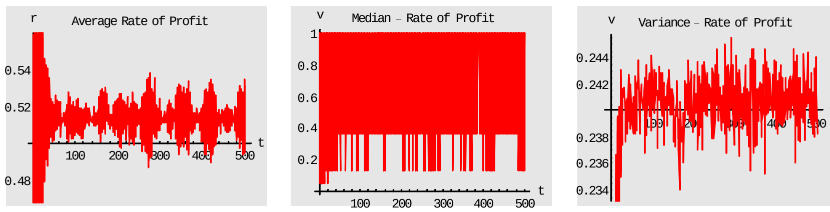

Figure 3.4: Rate of Profit - Descriptive Statistics: 7500 k

rate of profit over time does not mean that the cross-sectional variance showed the same behavior. Actually, quite the opposite happened, as we observe in the figures below. The skewness series wandered in general around zero with sudden jumps, which correspond to similar movements in the cross-sectional mean rate of profit. Kurtosis initially decreased, but as recursions proceeded it became stationary around 1.02, an even more platykurtic distribution than the one found in the cases when capitals were equal to the number of nodes.

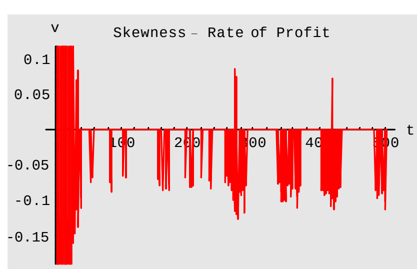

Figure 3.5: 7500k

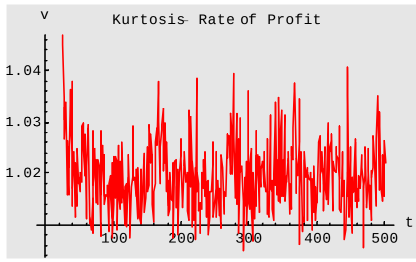

Figure 3.6: 7500k

Convergence of Variance: $K = 4N$

Finally, when we reached 10000 capitals (figs. 3.7 to 3.11), the patterns seen above change considerably. A strong convergence of the average rate of profit to very low ranges of variation after 200 recursions and the chaotic patterns of the preceding pictures disappear - a much more regular oscillating is observed in the mean series. Notwithstanding this fact, capitals still do not settle down to a totally equalized rate of profit and the most relevant feature is still the never ending gravitation of the series around a 50% rate of profit.

The dispersion statistics also show different characteristics. As a result of the convergence in the variance of the average rate of profit, skewness also converges to values around zero, but sudden jumps are not observed as often as in the immediately precedent case. Kurtosis, on the other hand, had a very clear exponential decrease to values close to one, with the difference in relation to the previous case that it did not seem to settle around a fixed value and seem to would have continued converging if more recursions were allowed.

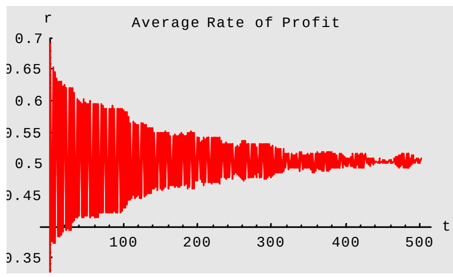

Figure 3.7: Rate of Profit - Mean: 10000 k

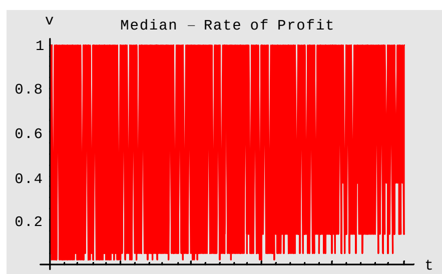

Figure 3.8: 10000 k

- $\alpha \leq -0.85, \kappa = 1$ - **Trajectories a la Schwartz-Sraffa: Fixed Point Attractor**

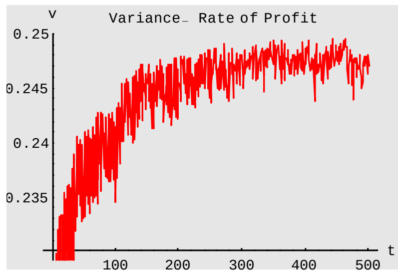

Figure 3.9: Rate of Profit - Variance: 10000 k

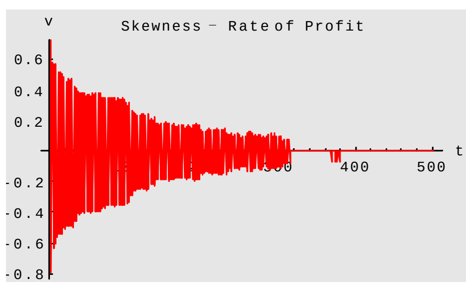

Figure 3.10: Rate of Profit - Skewness: 10000 k

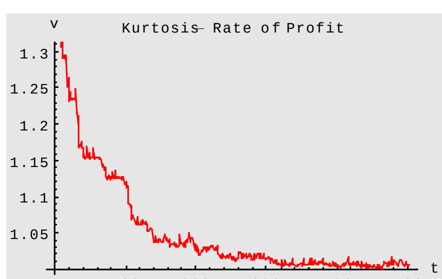

Figure 3.11: Rate of Profit - Kurtosis: 10000 k

After having found no sign that capitals would find a static equilibrium that could guarantee a single r under deterministic conditions , we calibrated the parameter of our model in order to find such result, by basically making individual capitals 'think twice' before they decided to move to another node. Fig. 3.12 show the result of the last ten periods of a 5x5 lattice in which equilibrium was reached. The series of descriptive statistics (figs. 3.13 to 3.15) help illustrate well the case. Shape statistics (not shown here) tended to infinite values as variance reached zero.

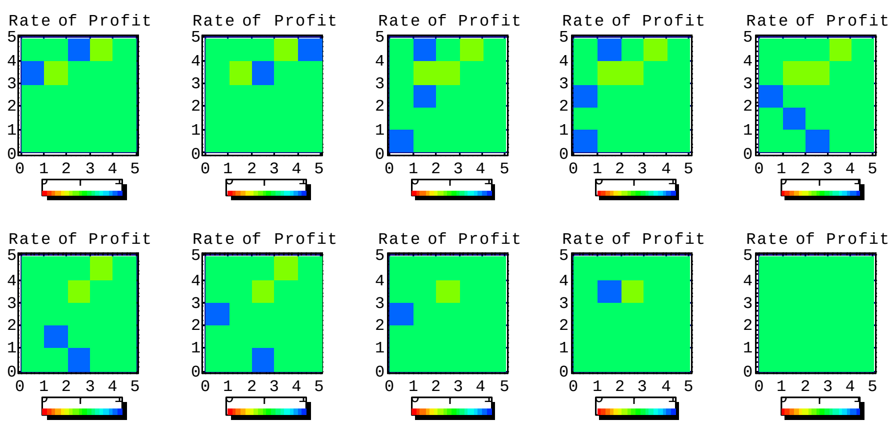

Figure 3.12: Fixed Point Attractor: Lattice

Convergence bore a non linear relation with parameters α, κ of the Probit function. For example, for the small matrix presented above, the convergence in the case $K = N$ with $\alpha = -1$ and $\kappa = 1$ was attained in 81 t. When $\alpha = -0.9$

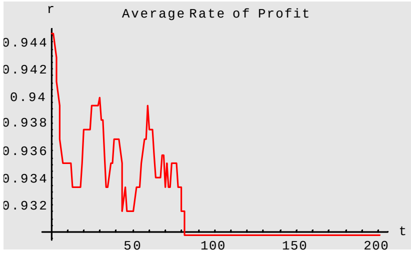

Figure 3.13: Fixed Point Attractor

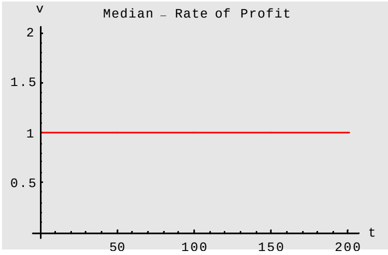

Figure 3.14: Fixed Point Attractor: rate of profit, median

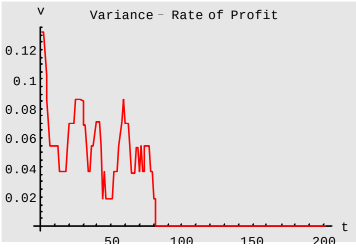

Figure 3.15: $K = N, \alpha = -0.85, \kappa = -1$

convergence is only reached after 241 recursions, when $\alpha = -0.85$, capitals settled down in 338t and when $\alpha = -0.83$, it never reached a fixed point after 500 recursions.

When we kept the same parameters of the probability function under which convergence was reached (again, $K = N$; $\alpha = -1$, $\kappa = 1$) and increased the lattice to our initial 50x50 dimension, capitals never settled down after 3000t and their statistics seem to suggest an even higher volatility than in the correspondent deterministic case (figs. 3.16 to 3.21).

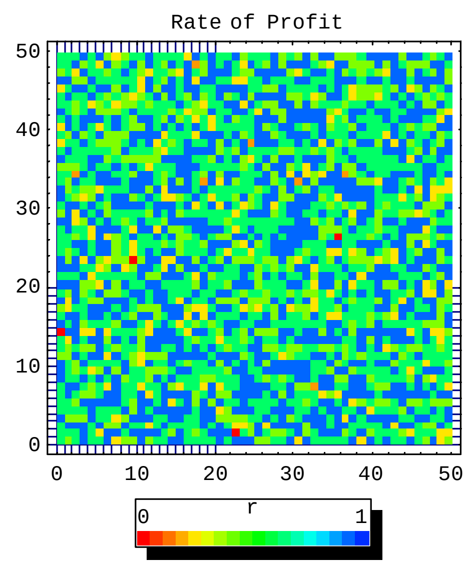

Figure 3.16: Rate of Profit distribution on a lattice-economy after 3000 t

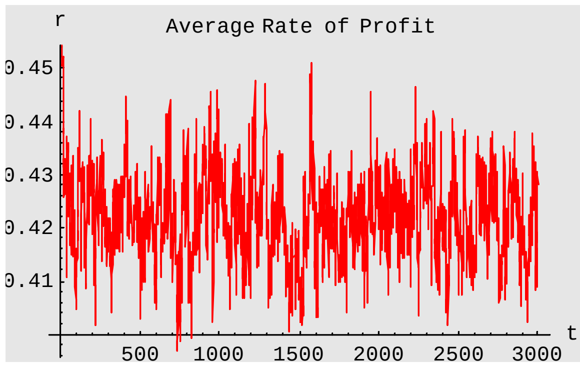

Figure 3.17: Rate of profit, 3000 recursions, K=N: average

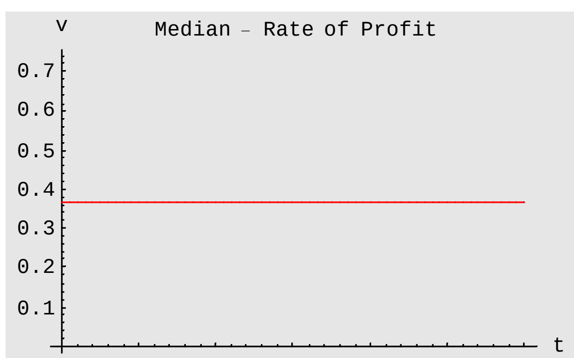

Figure 3.18: Rate of profit, 3000 recursions, K=N: median

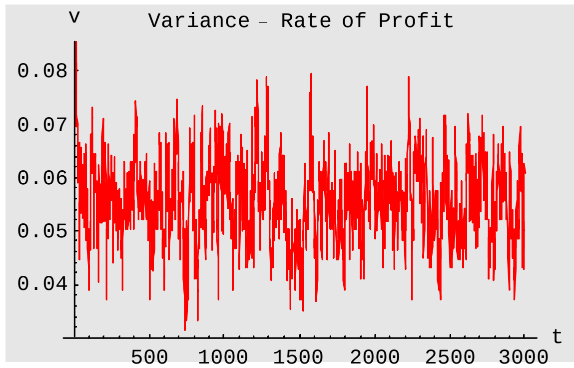

Figure 3.19: Rate of profit, 3000 recursions, K=N: variance

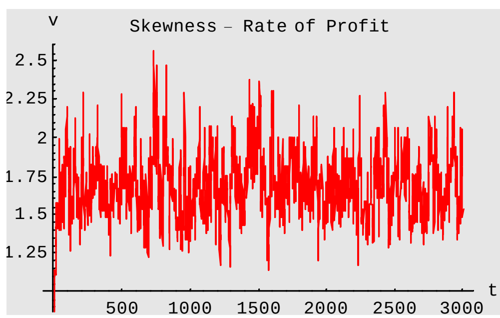

Figure 3.20: Rate of profit, 3000 recursions, K=N: skewness

Hence, convergence of the rate of profit to a fixed point was found only for some cases of our stochastic version of the model. Parameters of the probability function had to work as decelerators of the movement of capitals in order to avoid the constant grouping and dispersion of capitals in search for maximum profits,

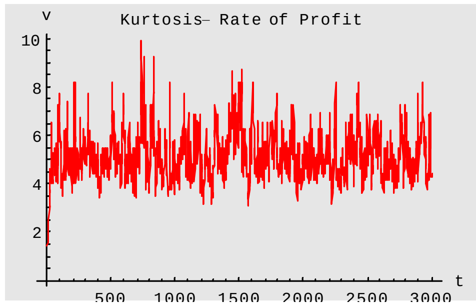

Figure 3.21: Rate of profit, 3000 recursions, K=N: kurtosis

but these equilibrium conditions were very precarious and depended largely on the total amount of capitals as well as on dimension of the lattice. The disequilibrium properties seem to easily override the artificiality calibrated equilibrium conditions.

3.1.2. Homogenous Capitals with Perfect Information

We also wanted to investigate the influence of the limited vision of agents on the convergence properties of the average profit rate by gradually extending the radius of the Moore neighborhood until it reached the entire lattice, a situation that corresponds to maximum vision by capitals. Figs. 3.22-3.32 show the results of the recursions of capitals with perfect information of the state of the economy under total capital compositions corresponding to the cases of capitals with limited vision analized above.

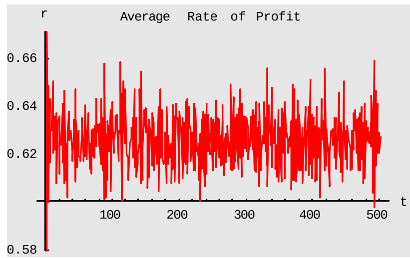

Figure 3.22: Rate of profit, Perfect Information, K=N: average

Figure 3.23: Rate of profit, Perfect Information, K=N: median

Figures above resemble very closely our previous results for capitals with limited vision. Convergence to a equalized rate of profit did not take place and the different amount total capitals changed considerably the patterns of movement of the average rate of profit. Although we expected that the volatility of the series changed considerably, it did not happened. However, there seemed to be a small difference in the velocity in which series reached a path with a stable pattern -

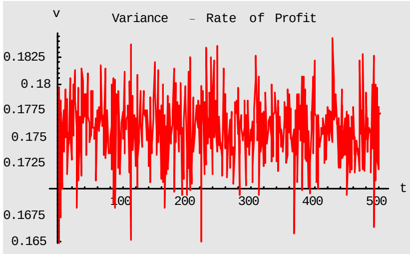

Figure 3.24: Rate of profit, Perfect Information, K=N: variance

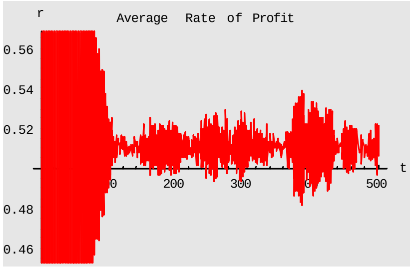

Figure 3.25: Rate of profit, Perfect Information, K=3N: average

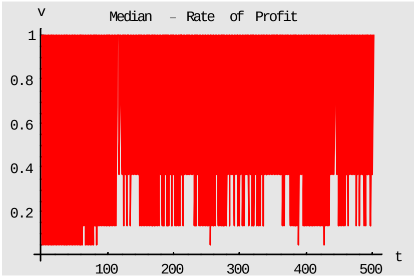

Figure 3.26: Rate of profit, Perfect Information, K=3N: median

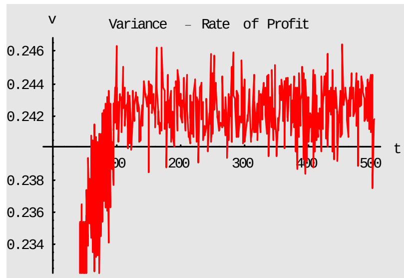

Figure 3.27: Rate of profit, Perfect Information, K=3N: variance

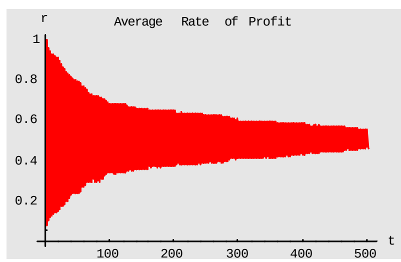

Figure 3.28: Rate of profit, Perfect Information, K=4N: mean

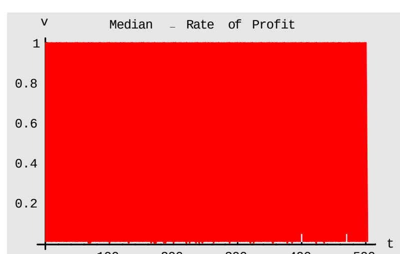

Figure 3.29: Rate of profit, Perfect Information, K=4N: median

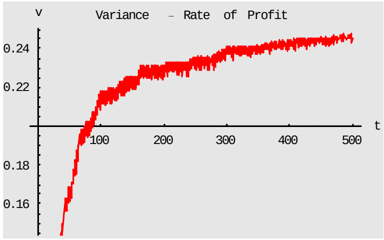

Figure 3.30: Rate of profit, Perfect Information, K=4N: variance

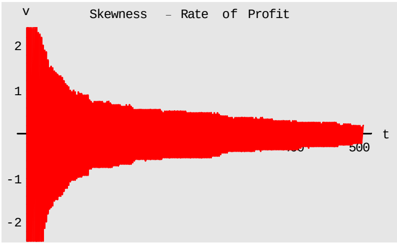

Figure 3.31: Rate of profit, Perfect Information, K=4N: skewness

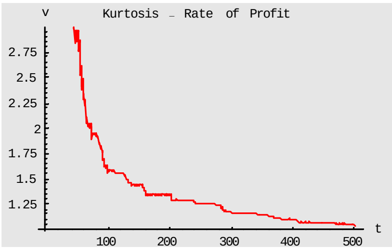

Figure 3.32: Rate of profit, Perfect Information, K=4N: kurtosis

in the cases here presented, it took more time for capital to overcome the initial conditions created by our random arrangement of capitals in the lattice.

3.1.3. Summing up:

So far, we have mostly described the patterns observed in the figures. But what do they mean? First, we believe that the most common characteristic in the cases analyzed above is that, in the period observed, capitals do not seem to settle to a fixed rate of profit. There is a continuous process of gravitation around some long run mean and for most cases, disequilibrium mechanisms did not allow a fixed point to be attained. If we were to talk about a long-run equilibrium, a more appropriate notion should be a quasi-steady state, where patterns of gravitation around a mean are established when we allow for enough recursions of the model.

A fixed point equilibrium was reached under very particular conditions, i.e., when the total number of capitals in the lattice was equal to the number of industries and the parameters of the probabilistic model were set to speed down the movements of capital.

When maximum vision is granted to the agents, results do not significantly change and the only perceived difference was on the speed of convergence to a oscillatory pattern.

The never ending turbulence observed in the great majority of cases is explained by the main dynamic feature of the interaction of individual capitals is created by differences in perceived and realized profit rates. Capitals move together into nodes with higher prospective rates of profit and this common decision creates forms of negative feedback that ultimately generate a cyclical pattern of attraction and repulsion of capitals.

Although this form of feedback explains in general the movements of the average profit rate, we have seen that as total capital increases, the volatility of the series considerably decreases. In our opinion, this is created by the relative 'freedom' of space enjoyed by capitals, combined with a smaller range of possible rates of profit in cases with less capital. Hence, as capital cluster and separate, variations in the average rate of profit are higher and more sudden.

As capitals grow in the lattice, the general mechanism of common movements of capital does not really change but the larger spectrum of rates of profit has a strong stabilizing effect in the movements around a long-run average rate of profit (fig. 3.33)

We can also observe other signs of the same process in the other descriptive statistics for most cases. The median is initially constant in cases where the number of total capitals is smaller or equal to the number of industries and as

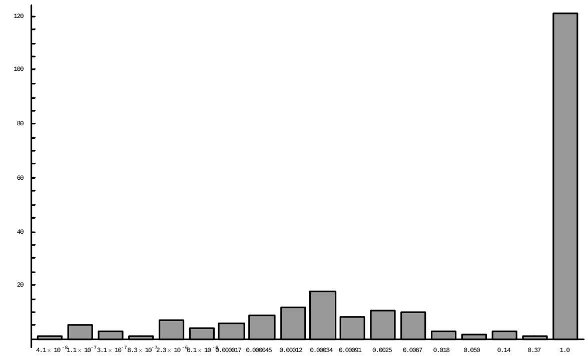

Figure 3.33: Histogram: Rate of profit, 200t, K=4N

capitals increase, median assumes a binary behavior. These are just different expressions of the same phenomenon - the clustering of capitals in the same nodes. In cases with few capitals the rearrangement of capitals did not influence the median and when capitals are abundant, a more diverse range of rates of profit allows changes in the overall frequency of rates of profit to be felt in the median.

The cross-sectional variance and the standard deviation of the rate of profit increased with the number of capitals and became clearly less volatile, thus reflecting movements of the mean. The movement over time of the cross sectional variance should not be confused with the variance of the time series of the rate of profit, which clearly diminished with the increase in the amount of capitals.

We should remark that changes in the volatility of the average rate of profit were created in most cases by a purely deterministic process, without resorting to random shocks, and still well determined and relatively constant structures could be identified by the descriptive statistics - a characteristic of many non-linear dynamic processes, especially of chaotic nature.

Finally, if we were to generalize the results here observed, it seems obvious to say that a necessary condition for the existence of a unique rate of profit of equilibrium is that the number of homogeneous capitals be equal or a multiple of the number of nodes in the lattice.[5]

3.2. What are the pre-asymptotic properties of the profit rate processes?

We now turn to the investigation of the statistical characteristics that emerge from the interaction of our individual capitals. We are interested in knowing through a set of tests what kind of behavior our artificial time series present and if it bears any resemblance with the essential properties of similar models that deal with competition in real markets.

[5] This conclusion, as simple as it may seem, is very similar to the point made, in another context, by F.Y. Edgeworth when criticizing the idea of a Walrasian geneneral equilibrium. Edgworth (1978) argued that the the existence of a equlibrium could only be proved when capitals are divisible, therefore assuming some form of 'jelly capital' that spread outs evenly around the sectors of the economy. This argument can be roughly translated into our case when $K = N$.

3.2.1. Stationarity Tests and Long-Run Autocorrelation

After looking at the series for the average rate of profit and its variance, and perceiving that as capitals grow in the lattice different patterns emerge in both series, a natural question to be posed is if these series present stationary properties. Figs. 3.34 to 3.37 show autocorrelation and partial autocorrelation functions for cases of the deterministic version of the model with different ratios between total capital and number of industries.

What we observe in the figures is that, as amount of capital grows in the lattice, the correlograms start to indicate elements of non-stationary in the series. The series, that initially presented no sign of autocorrelation, start to jump out of the confidence intervals in erratic fashion and finally establish a pretty well-determined pattern as we reach the amount of capitals equal to four times the number of nodes($4N$).

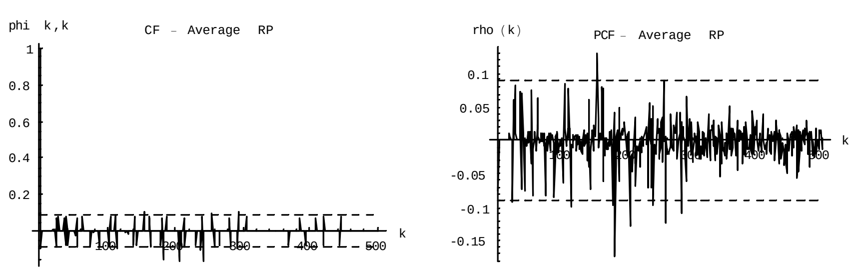

Figure 3.34: Correlograms: rate of profit, K<N

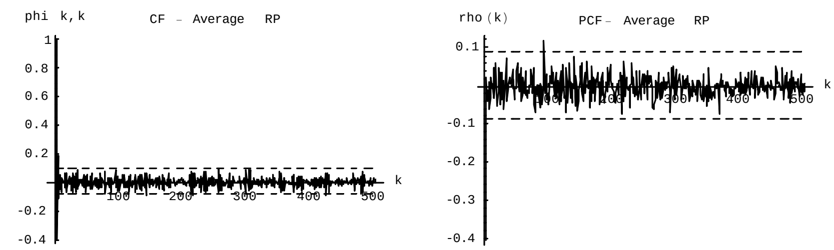

Figure 3.35: Correlograms: rate of profit, K=N

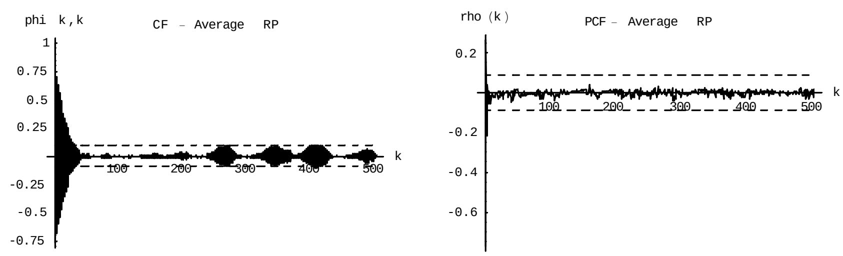

Figure 3.36: Correlograms: rate of profit, K=3N

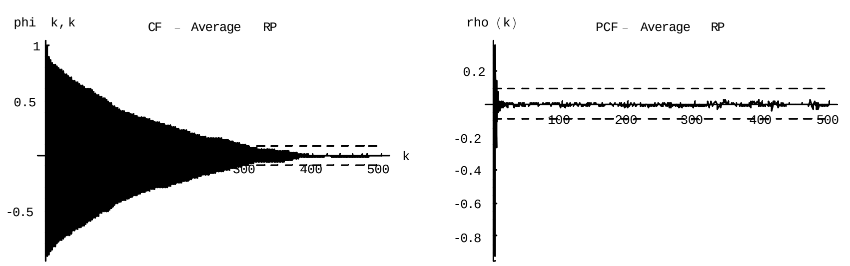

Figure 3.37: Correlograms: rate of profit, K=4N

The results shown above for cases $3N$ and $4N$ are signs of significant long range autocorrelation in the series. It seems that these systems have a long term memory, as pretty big lags still seem to have influence in the current state of the average rate of profit. Such statistical processes have been calling the attention of researchers, especially in financial markets, where absolute values of returns seem to exhibit the same characteristics - it decays according a power laws and seem to be to persist over large time scales, thus posing problems to standard econometric models which assume that the effect of past lags die out either rapidly or with the proper transformation of the raw data.

3.2.2. Skewness and Stable Distributions

These patterns in the autocorrelation call for the analysis of the probability distribution of the rates of profit, as models with long run autocorrelation have normally leptokurtic characteristics and so-called fat tailed distributions. The histograms below (figs.3.38-3.39) depict the results of 50 interactions of individual capitals. In the upper set of histograms we show the results the most characteristic form of frequency distribution found in our simulations. Distribution is negatively skewed, with a huge spike on the maximum rate of profit frequency and flat-topped, suggesting the existence of power-law distribution of profit rates.

This particular shape of the histogram is caused for two reasons: first, because we are not counting a range of small rates of profits in one single category, which aggravates the skewness to the right of the picture. Second, because we are considering prospective profit rates, which include a large number of nodes with a maximum rate of profit (i.e. rates equal to one). We believe this procedure is appropriate as it reflects the main mechanism behind (and result of) the dynamics of the model - continuous clustering of capitals in the same nodes and the consequent emergence of a high frequency of 'empty industries'. Notwithstanding this fact, if we were to correct for these two elements, the distribution would still be Stable with a change in the degree of kurtosis and sign of skewness, but not in its general shape.

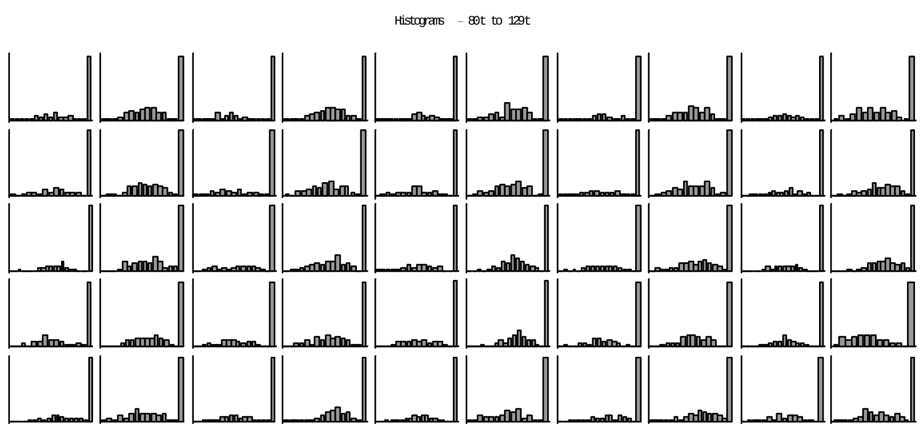

Figure 3.38: Histograms, rate of profit, K=3N: 80 t to 129 t

Contrasting with the figures above, we now show the histogram for the model in which a fixed point equilibrium was attained. As, we expected distribution presented a peaked form and was never particularly skewed. As it was mentioned before, this case was only found in very particular circumstances.

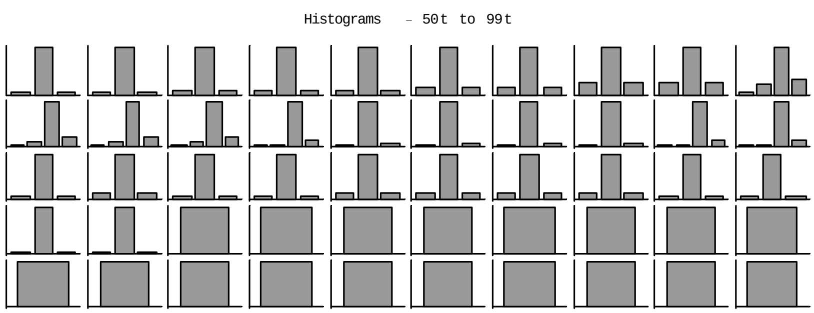

Figure 3.39: Fixed point rate of profit. K=N, with $\alpha = 1, \kappa = -1$

3.2.3. The General Rate of Profit as Zero Entropy Measure?

A Natural extension of this study is to look at the entropy levels of the average profit rate. The use of entropy measures here represents not only an alternative way to assess the statistical properties of the process we have been analyzing so far but also helps clarify very meaning of the usage of entropy measures in social sciences.

Amartya Sen (1973), for example, sees a contradiction between entropy being a measure for equality in economics and at the same time being a measure of disorder in thermodynamics and calls the Theil entropy measure as an 'arbitrary formula'.

The fact is, depending on the way we look at the frequency distributions we can have either equality as disorder (e.g. books spread out all over the floor) or the opposite - certainty and order (e.g. if we count the number of industries within the same rate of profit category or the choices made by people with the same tastes). Therefore, for us it seems that both concepts are valid, depending on the ordering process we are adopting.

We will be using as our measure of entropy the Kullback-Leibler distance which is defined by $H = \sum_{n} p_k \log_2 \frac{1}{pk}$, where p_k is the participation of firms in the k^{th} profit rate category and $H = 0$ for $p_k = 0$. Hence, an equalized rate of profit will mean zero entropy, implying zero uncertainty in the system.

Figs 3.40-3.42 present the results of our canonical examples. Some interesting properties were found. First of all, as the number of capitals increase, the volatility of entropy series increases considerably but, contrary to what is sometimes believed to be the behavior of closed system, entropy does not always increases with the number of capital. When we have $K = N$, the entropy series gravitates around 0.44 to 0.52, to jump in the case of $K = 3N$ to a range of gravitation between circa 0.55 to 0.67. Finally, when we reach K=4N, entropy falls to a rough average of 0.26. This sudden change in the behavior of the series when we move from $K = 3N$ to $K = 4N$, which was also found in the behavior of other series calls for closer investigation in future works.

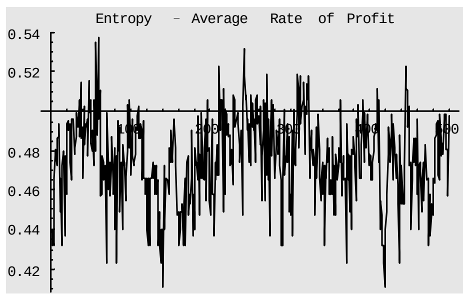

Figure 3.40: K=N

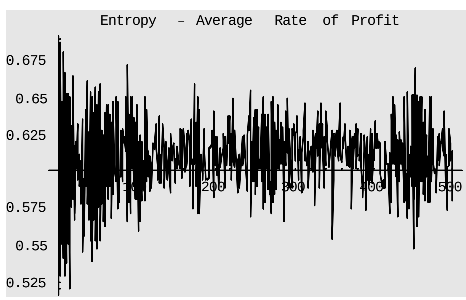

Figure 3.41: K=3N

As far as the equilibrium conditions are concerned, the figures 3.43-3.44 show the equalized rate of profit as zero entropy and the effect of the increase of the

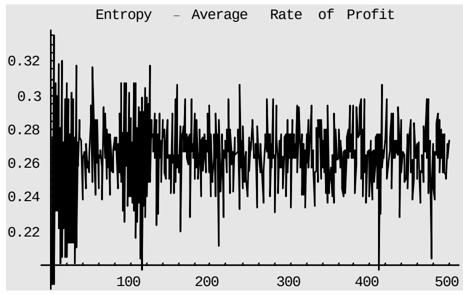

Figure 3.42: K=4N

dimension of the lattice on these equilibrium conditions. In the latter case, the impact of a higher number of industries is significant and the disorder in the system increases and does not seem to revert to minimum values

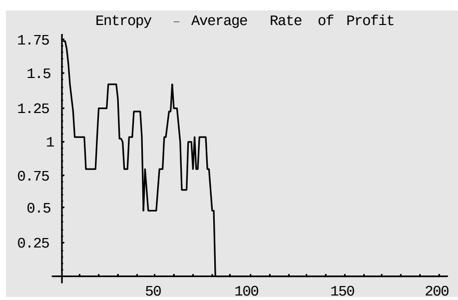

Figure 3.43: Equilibrium conditions (K=N, $\alpha = -1, \kappa = 1$, dimension=5x5)

3.2.4. Random Walks around the lattice

We have observed in all our results the effect of agents bounded rationality in creating clusters of capitals as information regarding prospective rates of profit

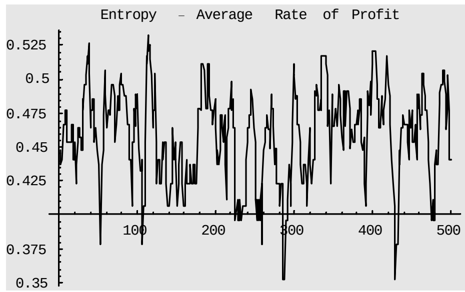

Figure 3.44: Equilibrium conditions (K=N, $\alpha = -1, \kappa = 1$, dimension=50x50)

does not match the realized rates of return. Hence, it would be interesting to look at how individual capitals move around the lattice under the conditions of free competition.

First, when we look at the behavior of the rate of profit in two industries over time, we can have a notion of how the gravitation of the general rate of profit occurs at the sectoral level. Fig. 3.45 suggests that in fact the movement of capitals across industries creates a continuous mismatch between individual rates of return.

When we observe how a single capitals moved around the lattice over the range of an entire simulation, the intuitive appeal of the Smithian competition is even stronger. Fig. 3.46-3.47 shows that although capitals are initially restricted to interaction around their own neighborhoods (given by the concentration of dots and color density in the lattice) , they start to move randomly around the entire

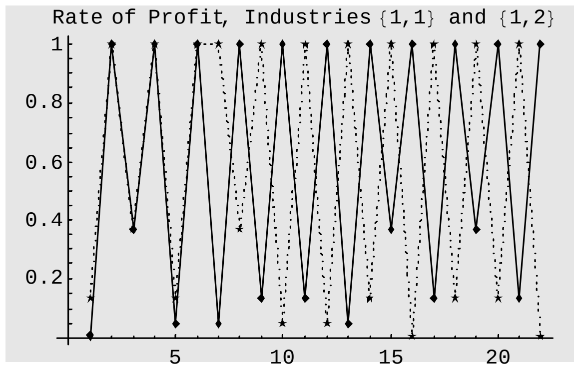

Figure 3.45: Sectoral average rate of profit

space of our artificial economy in a movement that resembles a random walk. Moreover, the number of different industries visited by a single capital seem to increase with the number of individual capitals in the lattice, demonstrating the powerful dynamic effect of the interaction of agents.

Conclusion

In this article we have modeled and simulated Smith's idea of the emergence of a general profit rate by using an agent-based methodology.

Our results show that capitals do not settle to an fixed point equilibrium that would correspond to a static, fully equalized rate of profit. In general, the result of

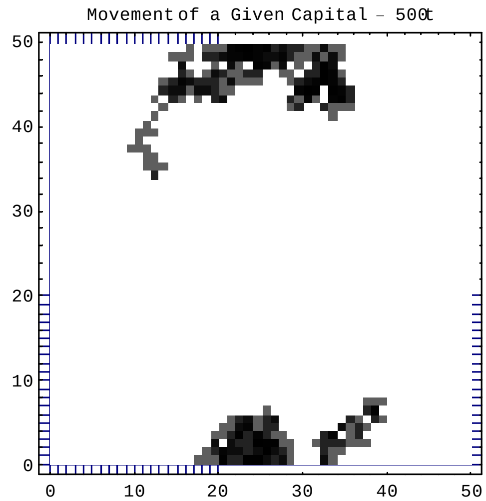

Figure 3.46: Random walks of individual capitals

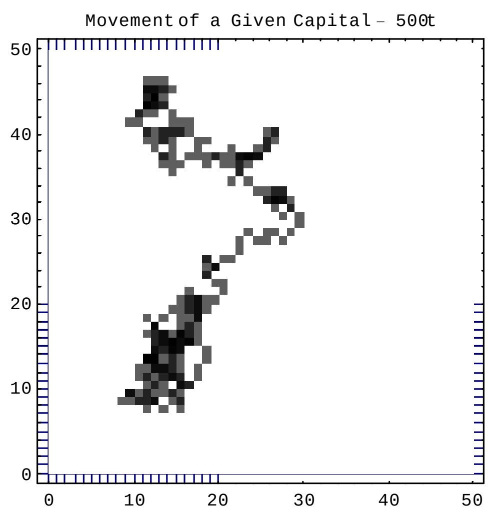

Figure 3.47: Random walks of individual capitals

capital interaction in our artificial economy is the establishment of a never ending oscillation process that becomes a gravitation around a mean of the series a the total number of capitals are increased in the lattice. This finding, we believe, sheds some light in the debate on the meaning of the general rate of profit in the work of the classicals and supports the idea that this concept is better represented by turbulence around a mean rather than any long run fixed point equilibrium.

The statistical process that emerges from the interaction of capitals in our models has characteristics that closely resemble properties also found in financial rates of returns in the real world: stable distributions, heavy tails, pronounced skewness and long term autocorrelations. Characteristics of chaotic behavior were consistently found in a range of parameters that corresponded to an specific proportion between nodes of the lattice and total number of (homogeneous) capitals.

We believe these results still leave many questions unresolved and call for further developments of the model. A closer look at its statistical characteristics and a sharper comparison with other arbitrage models can unveil the basic reason behind the departures from normality found in most of the aforementioned distributions. In addition, the incorporation of accumulation of capital seems to be the most natural next step in the research, as great part of the classical's insights on the effects of the increase of total capitals in the economy competition can only be poorly approximated with methods of comparative statics.

Finally, a look at the results here presented and the possibilities of further research shows that Smith's analysis of competition is indeed quite robust and fertile. We believe his work gave political economy the basic elements for the posterior development of an account of economics that is more realistic than the ones we find in modern mainstream and some heterodox traditions. The path opened by Smith, Ricardo and Marx can constitute the analytical core of a theory where competition - and not the lack of it - explains the dynamics of prices, technical change and distribution, without resorting to a vision of markets as decayed entities evolving from an imaginary age of perfect competition or to brute force appeals to stylized facts - and hence to explanations that aspire to explain regularities without being flexible enough to explain departures from them.

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