

## **Chapter 12. Conclusion: Theories and Models inspired by empirical regularities of Socio-Economic Spatial analysis.**

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### 12. 1. Introduction.

In this concluding chapter we will try to explain the fundamental role of empirical regularities of socio-economic spatial phenomena in the construction of theories in Spatial Analysis and Regional science. These empirical regularities help to construct new theories by developing new mathematical schemes, which not coming from exact sciences.

Last centuries of scientific enquiry were characterized by growing difference between the development of exact sciences (mathematics, physics, and chemistry) and their "soft" counterpart (socio-economic, socio-ecological, biological and behavioral sciences etc.). This difference essentially based on the application of mathematical methods. Theories of exact sciences are mathematically formulated and use methods of modern mathematics and computer science. The "soft" sciences avoid the use of mathematical language, but the processes of mathematization and computerization increasingly spread in their theoretical development.

The discrepancy between the theories of exact sciences and socio-economic sciences is the direct heredity of the separation and differentiation of sciences on previous millennia.

Let us compare briefly the forms of construction of the fundamental theories in Mathematics, Physics and Socio-Economic Sciences and Regional Science and Spatial Analysis

In mathematics from the times of Euclid (*ca.* 300 B.C.) till modern times the most effective method of construction of theory is the axiomatic approach. The mathematical theory usually starts from the description of mathematical objects and their properties (axioms). The nature of the axioms is not of importance; they should satisfy three necessary properties postulated by David Hilbert (Hilbert, 1899): consistency (i.e., impossibility to deduce from axioms a pair of two theorems which contradict one other), independence (this means that no axiom may be a logical consequence of other axioms) and completeness (i.e., it is possible to deduce from system of axioms either a proof of any proposition about elements of the system or a proof of the negation of that proposition).

The mathematical theory presents all possible logical consequences (theorems) of the system of axioms. Nevertheless, as stated the most prominent mathematician of XX century John von Neumann: "All mathematical ideas originate in empirics, although the genealogy is sometimes long and obscure."

The new age to mathematical science came with appearance of modern computer and development of computer sciences. The enormous computational power of modern computers transformed them from pure computational tool into the theoretical tool for the construction and justification of new mathematical theories, such as theory of cellular automata and artificial intelligence theories, theory of fractals, complexity theory, computer realization and visualization of non-linear dynamic processes and their critical bifurcation thresholds.

The construction of theories in Physics is much more complicated. The central notions of physical theories are "First Principles" or "Laws of Nature" which are the generalization and quintessence of empirical regularities of immeasurable amount of physical experiments. The logical consequences of these "Laws of Nature" leads to new experiments and new experiments leads to the reformulation and extension of "First Principles".

In the second part of XX century as a result of new prominent discoveries in physical sciences the changes in paradigms have taken place: repudiation of paradigm of inversibility of physical dynamic laws (the paradigm of equality of movement of the system in time – from past to future and from future to past) and the repudiation of Laplacian determinism (asserting that the knowledge of state of the dynamic system in some moment of time gives the possibility to define all its past states and predict all future states of the system). These paradigmatic changes occur because of transfer to the new study objects – the open developing systems whose states are strongly non-equilibrium states with respect of surrounding environments. The inversibility and non-equilibrium condition of open systems allow uneven jumping transfer of system into the states with higher level of order. The power, flexibility and richness of mathematical and physical methods and principles were the forces pushing towards the unification of theories in exact sciences and transfer of these methods and principles into the field of social, behavioral and biosciences. As continuation of this unification tendency in the second part of XX century several multidisciplinary meta-theoretical unifying approaches appeared such as General System theory (Bertalanffy, 1962, Rapoport, 1966), Cybernetics (Wiener, 1948),

Synergetics (Haken, 1983, see also Springer Series in Synergetics), Sociodynamics (Weldlich, 1983, 2000), Complexity theory (Haken, 1983, Prigogine, 1980, 1984, Cowan, Pines, Meltzer, 1994, see also Santa Fe Institute Studies in the Science of Complexity). Basic contribution of these approaches is directed "from model to reality" and not vice-versa. The weakness of such methods of construction of theories in social sciences was understood much earlier by Max Planck: "... *mathematicians, chemists and physicists often have a propensity to use their precise methods for explanation of biological, psychological and sociological events. But here it is necessary to ponder over whether the bridge constructed from ideas is robust enough, because only one of its supports is well reinforced*" (Max Planck, 1944).

Nevertheless, the successful extensions of the first principles of exact sciences to the global meta-theoretical principles appropriate to all scientific approaches continue. Examples of such successful extension in Socio-Economic sciences can be found in Synergetics of Hermann Haken and Sociodynamics of Wolfgang Weidlich. Synergetics and Sociodynamics grew from the physical theory of phase transition, namely from the mathematical operationalization methodology of LASER theory.

Synergetics departing from Physics doing out to study of systems composed of many subsystems of quite different natures and to search for principles which govern the processes of self-organization of these subsystems irrespectively of the nature. In Synergetics the macro-dynamical evolution of complex system dominated by a few key order parameters only. (Haken, 1983, see also Springer Series in Synergetics)

Sociodynamics in turn in principle is applicable to the modeling of dynamic phenomena out of different sectors of human society conventionally investigated in separate social sciences such as sociology, economics, demography and political and regional sciences. In Sociodynamics the dual causal relation between the micro level of inclinations, decisions and actions of individuals and the macro level of collective economical and political substructures and trends are taken into account. The main mathematical tool used in Sociodynamics is the Master equation – the system of differential equations of the continuous time Markov process for the probability distributions over the socio-economic variable based on the probabilistic transition rates per unit of time from one value of macro variable to its neighboring value (Weldlich, 1983, 2000, see also the chapter 6).

In the iteration processes the more general cases of the evolutionary dynamics of probability distributions over socio-economic variables are exists without the existence of stable Markov transition rates. Recently the new theory of non-linear discrete probabilistic chains was developed (Sonis, 2003, Dendrinos, Sonis, 1990, see chapter 7). The non-linear probabilistic chains are the generalizations of linear Markov chains in the case that set of transitional probabilities does not exists, but possible consider the dynamics of finite discrete probability distributions. The general probabilistic chains are generated by the iteration of transformations of the simplex of all probabilistic vectors into itself. The asymptotical behavior of non-linear probabilistic chains includes the bifurcation behavior (and bifurcation thresholds) much richer then the ergodic properties of Markov chains – quasi-periodic motion and different ways to chaos (Sonis, 1997b). The fine structure of three critical bifurcation thresholds is revealed: there are divergence and flip hyper planes; the saddle type flutter thresholds are described by the movement of  $m$ -dimensional simplexes (segment, triangle, pyramid, etc.) ( $m < n$ ) in  $n$ -dimensional space, and the orbits of discrete dynamics near the point on the flatter surface are presenting the movement on the surface of  $m$ -dimensional tori.

The different forms of probabilistic chains are useful for the statistical evaluation of relative dynamics of many socio-economic stocks, such as migration, population, capital, labor, etc. Important socio-economic interpretation of the probabilistic chains as discrete relative Socio-Spatial multiple population/many location dynamics is presented in the book (Dendrinos, Sonis, 1990).

The consideration of the non-linear probabilistic chains can be expanded to the consideration of the non-linear discrete barycentric chains in the Moebius space of all barycentric vectors.

## 12.2. First meta-theoretical Principles in Socio-Economic and Socio-Ecological Sciences.

In Socio-Economic and Socio-Ecological Sciences axioms or Laws of Nature are not formulated yet. In a very critical essay John Casti (1981) argued that the traditional modeling paradigm of exact sciences can not be employed for the most processes in social and behavioral sciences because of absence of natural laws in these sciences. Casti claims that "*in the absence of law the model must be constructed solely from the data by mathematical means*" (Casti, 1981, p.417).

Our book can be considered as an attempt to formulize and apply the general empirical regularities of social sciences whose methodological role is meta-theoretical and similar to the role of first principles in exact sciences. Next we will formulize following four meta-theoretical principles inspired by empirical regularities of socio-economic sciences: principle of collectivity, principle of complication, superposition principle and duality principle. The Ariadne thread which connected all these principles is the innovation diffusion theory in which innovation is treated as alternative of individual choice in human collectives (Sonis, 1981, 1983a,b, 1984, 1986, 1991, 1992a,b, 1997a, 2000, 2001.)

In the socio-economic and socio-ecological processes the innovation diffusion plays the quintessential role. Here we should stress the difference between invention and innovation. While invention involves the appearance of new information, innovation implies the spread of information [new or old] within the system. In other words, the innovation diffusion provides the mechanism of deepening of complexity (complication).

#### 12.2. 1. Principle of collectivity.

The principle of collectivity can be presented in the following form: each human being belongs to multiplicity of human collectives and using the knowledge accumulated in collectives. Collective (co-operating group of individuals) is characterized by its values, modes of choice behavior and the list of individual alternatives of choice. Collective imposes its values and modes of choice behavior of individual members of collective through action of innovators and innovating elites, (opinion leaders, different groups of choice makers i.e., different systems supporting, producing and spreading the innovations). In economics of capitalistic development of Schumpeter innovators appears as entrepreneurs (Schumpeter, 1943), in Political Sciences innovators are charismatic political leaders, in show business they are “stars” and “superstars”, in the Gumilev theory of ethnogenesis innovators are called “passionarii” (Gumilev, 1994), in the decision making and a political science they are demagogues and manipulators.

Therefore each individual member of collective has the ability to anticipate the choice behavior of other members of collective and partially evaluate public (collective) opinion of utility value of individual choice alternatives.

The mathematical presentation of this anticipation process leads to the log-linear system of differential equations describing the innovation diffusion process (see chapter 11 of this book). The log-linear system of differential equations based on the following formulation of the principle of collectivity: relative changes in choice frequencies depend on the distribution of choice alternatives between adopters of alternatives of choice. This hypothesis expends essentially the view point of the social statistical mechanics by including into the consideration the collective conscience of "human molecules", arising from the social interactions and informational mass media effects.

The principle of collectivity in analysis of behavior of human societies plays the analogical role as the Robert Hooke notion of cell in living organisms.

The principle of collectivity entails principle of complication.

#### 12.2.2. Principle of complication.

The complication means the deepening of complexity, i.e., the transfer from complex towards much more complex structures in the evolution of complex systems. The simplification means the clearing place for further complication by exclusion, reconstruction and destruction of less efficient substructures. The theoretical rationale of complication includes the study of the spread and partial adoption of new information that is characterized by a path-dependent process of self-organization within spatial socio-spatial complex systems. The paradigm of complication is pointing out on the deficiency of purely economic considerations of socio-economic systems and stresses the necessity to widen the concept of "Homo Oeconomicus" (Luce, 1959, Dreze, 1974) to the concept of "Homo Socialis" (Perroux, 1964, Sonis 1992b, 2000). Such a widening is radical in the study of complex socio-economic processes because of the important difference between the economic and socio-economic rationality: the traditional identification of economic rationality of "Homo Oeconomicus" as optimization is complimentary to socio-economic rationality of "Homo Socialis" as parsimony and risk aversion.

Let us compare briefly two types the socio-economic behavior presented with the help of models of "Homo Oeconomicus" and "Homo Socialis". Homo Oeconomicus is a totally egoistic, rational omniscient creature who is supposed to accomplish a rational free choice between different innovation alternatives on the basis of the utility maximization principle. Homo Oeconomicus knows a complete list of possible choice alternatives

knows their properties and spread abilities and evaluates rationally and in the exact form their utility properties He recognizes the universal form of utility function, which includes the factors of rational expectations. The choice behavior of "Homo Oeconomicus" based on optimization of unique universal utility function presenting the choice behavior of all agents of socio-economic system. Many extensions of this concept were elaborated on different grounds such as those of bounded rationality, satisfying behavior rules, habit consumption, choice and search routines, disequilibrium adjustment processes, etc. One can argue that all of these are presenting partially the behavior of Homo Socialis.

Homo Socialis is the "collective being", which cannot exist and survive without and outside of society (collective). He has no full information about all possible innovation alternatives, he do not know about the utility properties of these innovations and has no knowledge about the form of his utility function. The socio-economic behavior of "Homo Socialis" based on the learning process including the imitation and the knowledge about the factors of individual utility of choice alternatives gathered with the help of social interaction with other agents of system.

### 12.2.3. The principle of superposition.

The model of economic behavior of "Homo Oeconomicus" based on optimization of one utility function, and this optimization is a central idea of all modern economic analysis.

The model of socio-economic behavior of "Homo Socialis" based on existence of the several utility functions, presenting different extreme tendencies of economic development acting simultaneously, competing and complement each other in the evolution of socio-economic system.

The optimization of several utility functions is simultaneously impossible. The different utilities are realized only partially (with some weights) and the economic situation is presented by the superposition of different tendencies (weighted sum or resultant of tendencies).

The superposition principle is wide generalization of A. Weber principle of industrial location (Weber, 1909). The mathematical foundation of Superposition analysis is the Theory of Convex Polyhedra (Weyl, 1935).

(Minkovski-Caratheodory Theorem on Center of Gravity of convex polyhedron (Caratheodory, 1911)).

The applications of Superposition Principle are supported by development of the set of computer programs for practical implementation of Superposition Principle. This approach is systematically used for the analysis of states of systems in Mathematical Multiobjective Programming and in Migration Theory, Theory of Central places, Theory of Spatial Production Cycles and Production Fragmentation (Sonis, 1980, 1982; Sonis and Hewings, 1988; Sonis, Hewings and Okuyama, 2002 ).

#### 12.2.4. The duality principle.

Apparently, the first expression of the duality principle appeared in the XVII century in the Desarques Projective geometry in the form of Small and Great Duality principles (Poncelet, 1865). The Small Duality principle acts for points and straight lines on the plane. Each statement about points and straight lines and their incidence generates the dual statement by exchanging the word "point" by the words "straight line" and vice-versa. The Small Duality principle claims that if the initial statement is true then the dual statement true simultaneously. The Great Duality principle acts for the points, straight lines and planes and their incidence within three-dimensional space, but it is based on the exchange: "point"  $\Leftrightarrow$  "plane". In both these principles there are two systems of geometrical objects and their incidence and the duality rules which connect each object from one system with the dual object of the other.

The idea of duality penetrates all branches of mathematics (Bourbaki, 1960) and at present penetrates the field of application in socio-economic sciences.

Linear Programming gives the most impressive example of duality which interconnects two dual sides of production process: the physical production of goods itself and the pecuniary evaluation of this physical process with the help of "shadow prices" of Kantorovitch and Dantzig (see Dorfman, Samuelson and Solow, 1958).

It is important to note that Linear Programming is equivalent to the theory of Antagonistic Zero-sum games (Von Neumann and Morgenstern, 1953) in which the dual structures present the economic behavior of antagonistic players

In the location optimization models the shadow prices play a role of differential rents (Samuelson, 1952, Stevens, 1961).

In Migration theory and in the tourism analysis the well-known Push-Pull principle is based on duality connections between origins and destinations (see Sonis, 1980).

In Input-Output Key Sector analysis the backward and forward Rasmussen-Hirshman linkages between economic production sectors represent the dual structures in Input-output analysis. (Sonis, Hewings and Guo, 2000).

In Micro-morphology of arid desert soils the density of mineral layers in the soil present the dual connections with regime of precipitation in the desert.

The new example of the application of duality principle is the duality of interconnections between the choice behavior of an individual facing a set of choice alternatives and the competitive behavior of alternatives themselves (see Chapter 11 of this book). The choice behavior of individual is the central object of Dynamic Utility choice models, while the antagonistic competition between choice alternatives is the main point of diffusion of alternative innovations. Analytically the duality between theory of Dynamic Utility choice and Innovation diffusion theory lies in the fact that vectorial differential equation of the diffusion process has solutions which are the choice probabilities for the discrete choice models. Namely, the dynamic multinomial Logit model of utility choice is the solution of the system of log-linear system of differential equations for totally antagonistic innovation diffusion process; in the same manner, the dynamic multinomial Dogit model is the solution of system of differential equation for innovation diffusion process within active environment with environmental socio-ecological niches, preserving different innovation alternatives (Sonis, 1984, 1986).

The methodological meaning of duality principle can be presented as a transfer of ideas from the depth of understanding of one methodological approach to the depth of understanding of another. The duality means that, despite the different interpretations, the mathematical models generated through the various approaches are analytically similar. Each approach is associated with a different methodological base relating to the behavior of “Homo Socialis”. In this study, the mathematical description of the complex behavior of “Homo Socialis” in choice processes within

the collective is based on four different approaches, which give the same mathematical form to the innovation diffusion process in real space-time.

These approaches are (i) empirical regularities of the choice process – the S-shaped change in the portion of adopters of alternative innovations; (ii) the first principles of parsimonious human behavior as collective beings, (iii) the competitive behavior of social elites in the mathematical form of variation principles and (iv) the “lock in” captivity phenomenon in the behavior of social elites.

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